Lesson 35: Kepler's Three Laws of Planetary Motion

Kepler took the data that Brahe had spent his life collecting and used it (especially the information on Mars) to create three laws that apply to any object that is orbiting something else.

- Although Kepler’s math was essentially wrong, the three laws he came up with were correct!
  - It would be like you writing a test, and even though you did all the work on a question wrong, you somehow get the correct final answer.
- Kepler’s Three Laws of Planetary Motion are still the basis for work done in the field of astronomy to this day.

Kepler’s First Law

Kepler’s First Law went against scientists' major assumption at that time about orbits… in fact it is probably against the image of orbits that you have!

- If I asked you to describe or draw a sketch of the Earth’s orbit around the sun, how would you draw it. Think about in your head.
- You’d probably stick the sun in the centre and draw a circle around it to show the path the Earth takes.
- In fact, this is totally wrong, as **Kepler’s First Law** states…

“The path of any object in an orbit follows the shape of an ellipse, with the orbited body at one of the foci.”

So what does all that mean?

- An **ellipse** is shaped like a circle that someone has sat on. It’s squished in the middle and bulges out on the ends.
- **Foci** (the plural form of the word **focus**) are two points inside the ellipse.
  - If you were to push stick pins into the foci and put a loop of string around them, you could draw an ellipse.
- This means we have a shape that looks like this…

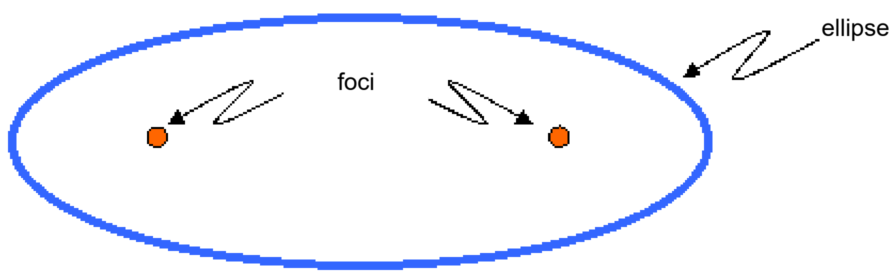

Illustration 1: An ellipse.

Remember that the object being orbited sits at one focus, and the object in orbit follows the path of the ellipse.

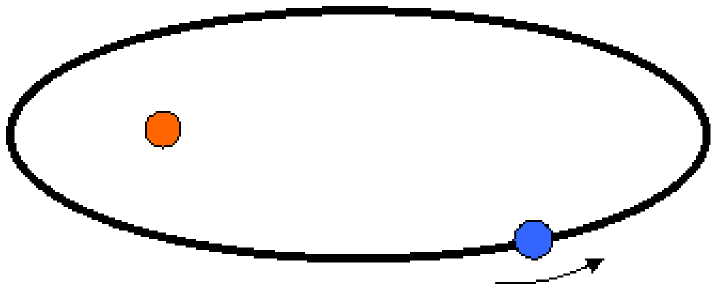

Illustration 2: The Earth orbiting around the Sun.

- This means that sometimes the Earth is closer to the Sun, and sometimes further away.
  - This is not the reason for summer and winter!
  - The seasons on Earth are created by Earth's tilt on its axis.

The amount of "stretch" of an ellipse is measured as the **eccentricity** of the ellipse. Eccentricity has a value between 0 (a circle) to 1 (a parabola).

The diagrams I have drawn here are exaggerated quite a bit to show the elliptical shape and focus clearly.

- The true orbits of the planets are not this stretched out.
- Most of the planets orbiting our sun have orbital eccentricities just barely over zero.

This elliptical shape does not just apply to planets orbiting the sun. It works for ***any*** object orbiting any other object.

- If you measured the orbit of the Moon around the Earth, it would have an elliptical shape, and so would any satellite in orbit around the Earth.
- Since orbits around the Earth are quite small, their shapes are almost a circle with an eccentricity of almost 0.

DID YOU KNOW?
Newton was able to show that his laws of gravitation gave the same results as Kepler's. In fact, Newton was able to take things farther with his strong math background to show that the shape of the orbits were conic sections (for those of you that have seen that stuff in math). We'll be looking more at Newton's contributions later.

Kepler's Second Law

Kepler's Second Law is based on the speed of the object as it orbits.

- In the Earth-Sun example shown in Illustration 2, the Earth will travel faster and faster as it gets closer to the sun.
- As the Earth moves away from the sun, it will move slower and slower.
- It's almost like the Earth is doing a "slingshot" around the sun very quickly as it passes near it.

Kepler didn't talk about speed when he wrote out his Second Law. Instead, he looked at a mathematical detail that pops out because we are talking about ellipses.

"An imaginary line from the sun to the planet sweeps out equal areas in equal times."

If we were to look at the area the Earth sweeps out in a 15 day period, first when close to the sun...

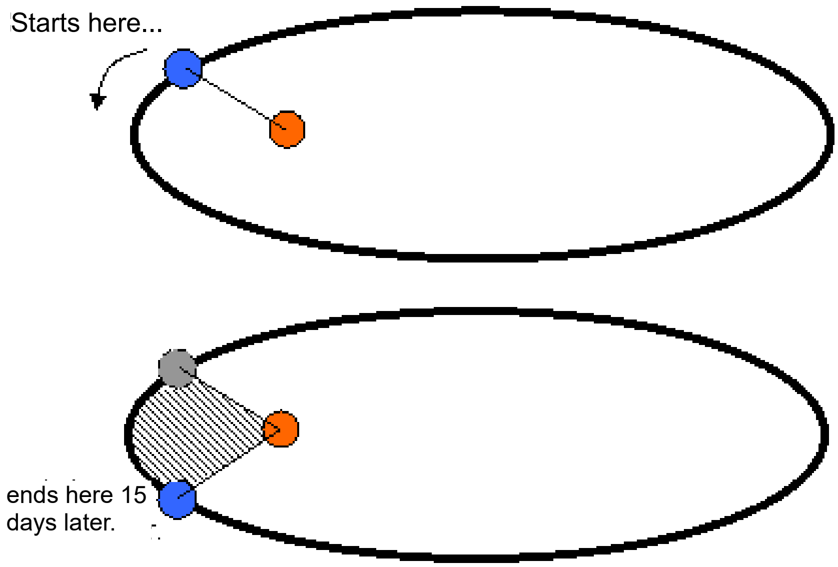

Illustration 3: Near the sun...

and then when far away, we get an idea of what Kepler meant.

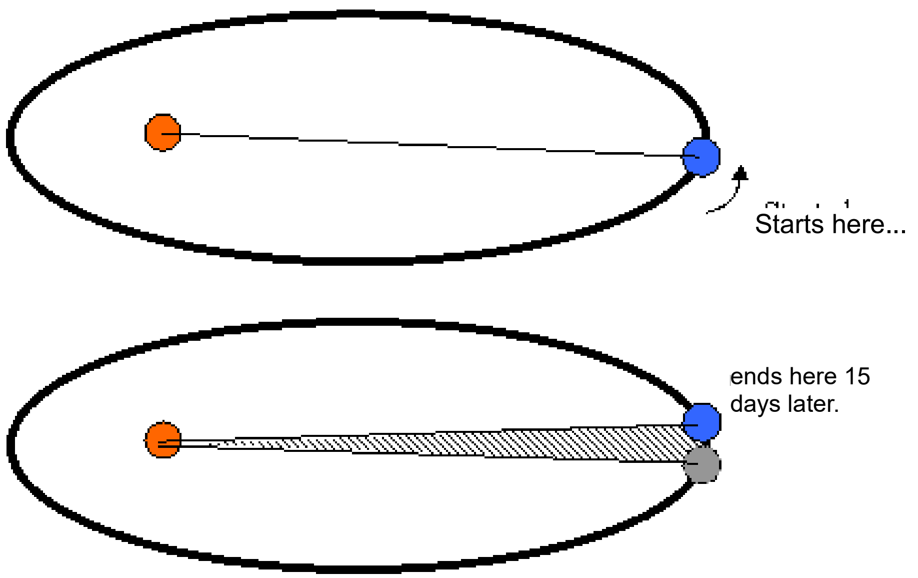

Illustration 4: ...and far from the Sun.

- Notice how in Illustration 3 we have a stubby, fat, (basically) triangular area that was swept out by the line, but in Illustration 4 we have a tall, thin, (basically) triangular area swept out.
- If we calculate the area that I have (more or less) shaded in as triangles, you would find that they are equal.
- This just shows that the planet is moving a lot faster when it is closer to the sun, since you can see that it traveled a greater distance along its orbit during that time.

Kepler’s Third Law

The big mathematical accomplishment for Kepler is in his **Third Law**, where he relates the radius of an orbit to it’s period of orbit (the time it takes to complete one orbit).

“The square of the period of orbit,
divided by the cube of the radius of the orbit,
is equal to a constant (Kepler’s Constant) for that one object being orbited.”

The formula looks like this...

$$K=\frac{T^2}{r^3}$$

T = period (in any unit, usually seconds)
r = radius (in any unit, usually metres)
K = Kepler's Constant

There's a few weird things about this formula compared to many other physics formulas:

- You can measure the period and radius in any units you want, as long as you keep them consistent for the whole question. I'd still suggest you use standard units whenever possible.
- The radius is the *average* radius of the orbit.
    - When Kepler did his original calculations he assumed a *circular* orbit in his calculations, even though this went against his own First Law.
    - Today we take the distance of the semi-major axis of the ellipse as the average radius of the orbit. These are the values for radius of orbit you'll see in most books and websites.
- Kepler's Constant is only a constant if the object being orbited stays the same.
    - So, anything orbiting the Sun (Mercury, Mars, Earth, Jupiter, comets, etc.) has the same Kepler's Constant.
    - If we change to things orbiting the Earth (the Moon, International Space Station, satellites), we will get a *different* Kepler's Constant than the one we got for stuff orbiting the Sun. The Sun's Kepler's Constant and the Earth's Kepler's Constant will be different from each other.

Example 1: Based on the values on the following table, **determine** the value of Kepler's Constant for objects orbiting the Sun. **Explain** the significance of the values you obtain.

Planet	Period (days)	Mean Radius of Orbit (m)
Earth	365	1.49e11
Mars	684	2.28e11
Jupiter	4331	7.78e11

“Mean” is just another way of saying “average.” This is the measurement this is based on the semi-major axis of the orbit of the planet.

For each planet's data, we will need to use the formula to solve for K. The values we would get are...

Planet	Kepler's Constant (d^2/m^3)
Earth	4.03e-29
Mars	3.95e-29
Jupiter	3.98e-29

You can see that all three planets give us approximately the same value for Kepler's Constant. This confirms that Kepler's Third Law is correct in predicting that there is a constant ratio between the squared period and cubed radius of objects orbiting the same object. We could take the average of these three values (**3.99e-29 d^2/m^3**) to get our final answer.

A better way to obtain this average value for Kepler's Constant would be to graph a linear regression of Period Squared as a function of Radius Cubed. The slope of the line of best fit would be Kepler's Constant.

Example 2: Neptune has an mean radius of orbit of 4.5e12 m from the Sun. Using the average value for Kepler's Constant that was calculated above for Earth, Mars, and Jupiter orbiting the Sun, **determine** how long it takes Neptune to complete one orbit.

$$K = \frac{T^2}{r^3}$$
$$T = \sqrt{K r^3}$$
$$T = \sqrt{3.99\text{e-}29 (4.5\text{e}12)^3}$$
$$T = 60298.32087 = 6.0\text{e}4\, days$$

Warning!
In most tables in books or websites you will see two radius values recorded for each planet. One is the radius of the planet itself (from its centre to its surface). The other is the mean radius of its orbit around the sun. Make sure you use the right one for the problem.

Notice that since this value of Kepler's Constant was calculated using days, our answer is in days. If we convert it into years (*annum*) we find that it takes Neptune 165 years to orbit the Sun once!

You can also write out this formula a couple of other different ways.

- They are just ways of rearranging things so you can do certain questions faster, or to let you cancel out stuff to make other questions possible to calculate.
- We know that Kepler's Constant will be the same for objects orbiting the same thing.
  - It would be like doing one set of calculations for Earth and another for Mars (like we did in the table above). The K values are the same, so we can just stick their formulas together.

$$\frac{T_e^2}{r_e^3} = \frac{T_m^2}{r_m^3}$$

e = values for Earth
m = values for Mars

- Using a little cross multiplying, this version of Kepler's Third Law is often re-written to look like this…

$$\frac{T_e^2}{T_m^2}=\frac{r_e^3}{r_m^3}$$

- You don't have to have "e" and "m" in the formula. I just used those because we were talking about Earth and Mars in the last example.

- You could just as easily use "a" and "b" (or whatever!), just as long as you put both the "a" on top, and both the "b" on the bottom.
- You can use this version of the formula to do tricks if you are missing some information, as the next example shows.

Example 3: If the orbit of Mars is 1.52 times greater than the orbit of Earth, **determine** how many days it takes Mars to complete one orbit.

If I was able to look up the orbit of Earth in a book, this would be a fast question. All I would do is multiply that number by 1.52 to get the orbit of Mars, and then I would have both the radii I need. But what if I was doing this question on an exam and I didn't know the value for Earth's mean radius of orbit? Here's what we do know…

$r_m = 1.52\ r_e$

$T_e = 365$ days

Let's see if we can substitute that into the formula…

Warning!
One last warning on this (it's such a common mistake). Do not use the "Radius of Earth" number on your data sheet. That is the distance from the centre of the Earth to the surface of the Earth, not the size of Earth's orbit. around the sun.

$$\frac{T_m^2}{T_e^2} = \frac{r_m^3}{r_e^3}$$

$$\frac{T_m^2}{365^2} = \frac{(1.52 r_e)^3}{r_e^3}$$

The radius of Earth's orbit (r_e) cancels out.

$$\frac{T_m^2}{365^2} = \frac{1.52^3 r_e^3}{r_e^3}$$

$$\frac{T_m^2}{365^2} = 1.52^3$$

$$T_m^2 = 365^2 (1.52)^3$$

$$T_m = 684.0033778 = 684\, days$$

Homework

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p275 #1