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Kepler's New Astronomy

by John F. McGowan, Ph.D for Math-Blog.com

Introduction

This year (2009) is the 400th anniversary of the publication of Johannes Kepler's book *New Astronomy (Astronomia Nova)* announcing the discovery of the elliptical orbit of Mars to the world. The discovery of the elliptical orbit of Mars and the mathematical rule of motion for Mars on its elliptical orbit by Johannes Kepler in 1605 is one of the most important advances in astronomy, physics, and science. This discovery transformed the unproven heliocentric theory of Copernicus into a rigorous predictive theory that outperformed the traditional geocentric theory of Claudius Ptolemy and his successors. The discovery paved the way for Newton's theory of gravitation. It remains one of a small number of cases where a simple mathematical rule for seemingly complex and confusing data has been found. In many respects, the discovery of the elliptical orbit of Mars and other planets is more important than the better known work of Kepler's contemporary Galileo. In honor of Kepler, NASA has named its recent mission to look for extra-solar planets, especially possible other Earths that might support life or even intelligence, the Kepler mission.

In Kepler's time the reigning Ptolemaic theory could predict the position of Mars to within a few degrees, usually less than a one percent error. How important is such a small error? Space missions routinely depend on modern orbital dynamics, a lineal descendant of Kepler's work, to make far more accurate calculations to succeed. The Mars Climate Orbiter mission in 1999 failed due to a tiny error. After traveling about 300 million miles, the Mars Climate Orbiter came in about 90 miles, a tiny fraction of 300 million miles, too low, burning up in the Martian atmosphere rather than aerobreaking successfully into orbit. Successful space missions, the Global Positioning System (GPS), and other modern applications depend on precision mathematical models similar to and sometimes directly descended from Kepler's model of the orbit of Mars.

Kepler's story is very different from the story of Galileo and it offers different lessons for today. Diverse fields ranging from astronomy and space physics to artificial intelligence are confronted with similarly complex and confusing data. A mathematical solution to an outstanding problem comparable to Kepler's discovery could reveal long suspected connections between gravity and other forces, perhaps enabling new power or propulsion systems, enable computers to recognize objects and spoken words, or solve other problems. This article will discuss the discovery of the elliptical orbit of Mars in the context of Kepler's time. It will also draw some lessons from Kepler and compare and contrast Kepler's process of discovery to modern astronomy, physics, space science and engineering, including a detailed discussion of dark matter and dark energy.

The Setting

Johannes Kepler was born on December 27, 1571. He was a contemporary of William Shakespeare and Galileo Galilei. Kepler lived in a tumultuous and difficult time plagued by numerous wars, revolts, and outbreaks of illness. Kepler was a Lutheran and repeatedly encountered difficulties due to his refusal to renounce the Lutheran faith. He was caught in the midst of a growing conflict between Catholics and Protestants. Although Kepler is often described either as an astronomer or a mathematician, which is technically correct, he was in fact an astrologer. Astronomers and mathematicians in the sixteenth and early seventeenth century were nearly always astrologers and often supported themselves, as Kepler did, by a mixture of teaching and predicting the future. In 1600 the Catholic Counter-Reformation forced Kepler to leave Graz (now in Austria). He was deprived of his teaching position because he refused to convert to Catholicism. Fortunately for Kepler, he was able to find a position as an assistant to the astrologer and alchemist Tycho Brahe, one of the many astrologers and alchemists employed by the Holy Roman Emperor Rudolf II in Prague.

The Holy Roman Empire was a bewildering confederation of mostly German-speaking city-states, duchies, and kingdoms. The Holy Roman Emperor was elected by a council of nobles and officials of the various states that comprised the Holy Roman Empire. In practice, the Holy Roman Emperor was often a member of the Habsburg royal family which dominated the Holy Roman Empire. Rudolf II was a Habsburg. At the time the Habsburg family had a long history of serious mental illness. Rudolf II was probably mentally ill.   He sometimes behaved oddly and eventually suffered what sounds like a serious nervous breakdown. Amongst other things, Rudolf II was fascinated by alchemy, astrology, magic, and similar topics, in many respects the "science" of the time. He sought to unlock the secrets of the universe. He invited hundreds of alchemists, astrologers, magicians, and scholars to Prague, employing a vast army of geniuses, mountebanks, and kooks in a grand effort to penetrate the mysteries of creation. His obsession took him dangerously close to heresy and perhaps even black magic, to the alarm of the Catholic Church and his royal Habsburg siblings and cousins.

Rudolf II and Tycho Brahe's interest in the orbit of Mars was far from idle academic curiosity. Rather they believed that Mars influenced the course of events on Earth. If they could learn to predict the future motion of Mars exactly, then they believed they could predict the future with greater accuracy through the practice of astrology. Kepler shared these views. In his pursuit of the exact prediction of the position of Mars, Tycho Brahe built a state of the art observatory and fabricated sophisticated instruments similar to sextants to measure the position of Mars more accurately than ever before. The telescope lay only a few years in the future. Tycho Brahe collected the most accurate measurements of the position of Mars to date but he lacked the mathematical skill to analyze the baffling data.

Kepler's time is often portrayed as the time of a sharp break between the newborn modern science and medieval religion and superstition, especially in accounts of Galileo and his conflict with the Catholic Church. The story of Tycho Brahe and Johannes Kepler is very different from this popular image. In many respects Tycho Brahe is a precursor of the modern observational astronomer and Johannes Kepler is a precursor of the modern theoretical physicist. However, Rudolf II, Tycho Brahe, Kepler, and their contemporaries looked at the world very differently from the "clockwork universe" of modern science. In many respects, they saw the universe, usually meaning the solar system, as a living system in which angels or intelligences directed the motions of the planets and indeed the entire universe. Most importantly for scientific and technological progress, Brahe and Kepler and many others believed that the universe followed precise mathematical laws, presumably established by God, either all or most of the time, not because the universe was a giant mechanical clock, but rather because the angels or intelligences followed the laws much as people obey laws.

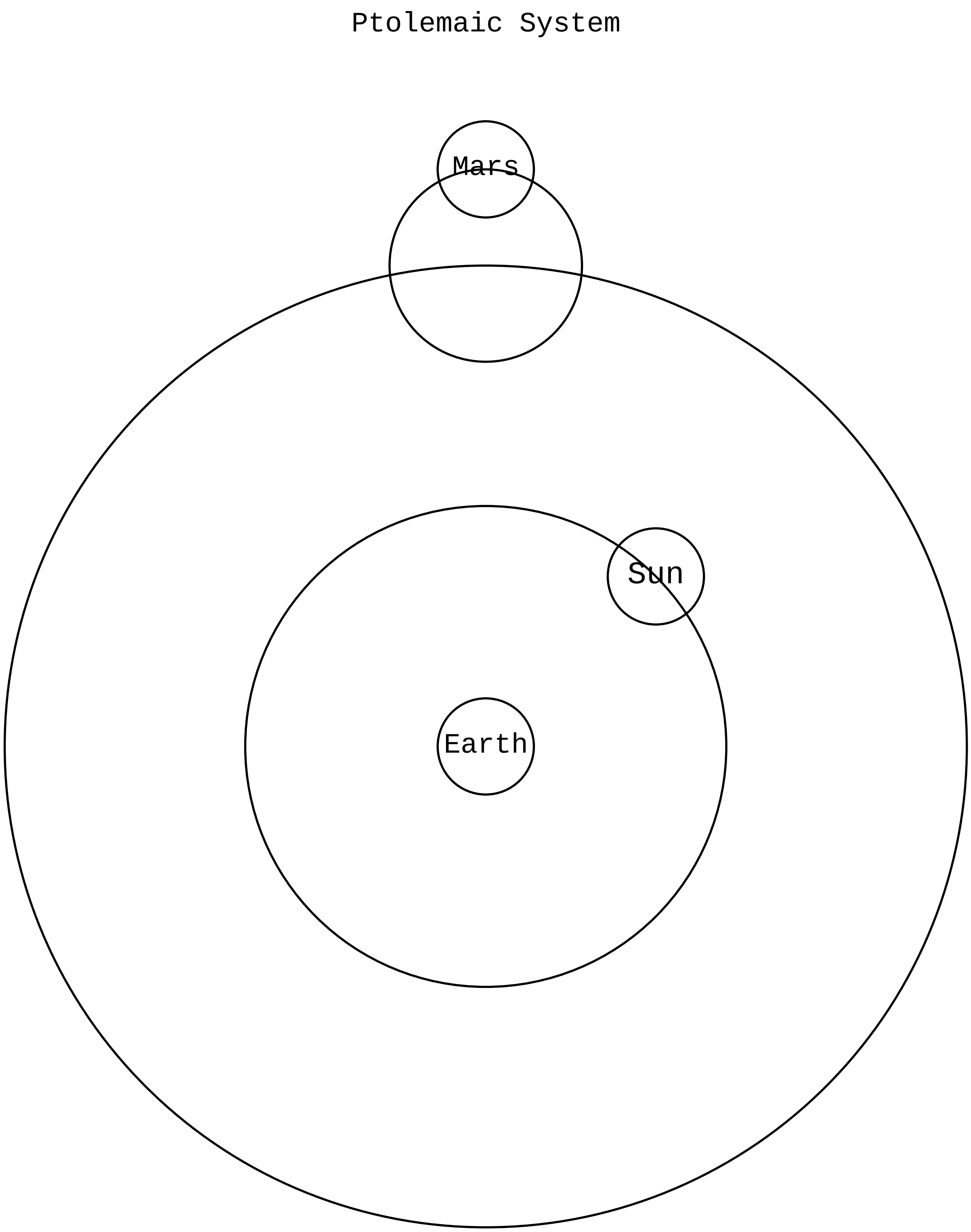
Three Theories

In Kepler's time, the prevailing view was that the Earth was a sphere about eight-thousand miles in diameter (the correct value) at the physical center of the universe which was what we would call the solar system. The seven planets -- meaning the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn -- orbited around the Earth. This basic theory was quite old. It probably dates back to ancient Sumeria (modern day Iraq). It is found in Plato's *Timaeus*, Aristotle's *On the Heavens* (*De Caelo*), and Claudius Ptolemy's *Almagest*. The ancients envisioned the planets as beings -- gods, intelligences, or angels -- that moved in perfect uniform circular motion. Plato and other philosophers assigned great importance to uniform circular motion. Plato's *Timaeus* contains language that suggests that Plato believed thought itself could be explained through circular motion; he may perhaps have been thinking of some ancient clockwork computational device like the Antikythera Mechanism. In Plato's time, wheels may well have been exciting high technology analogous to computers or space travel today. Plato and his followers were fascinated with mathematics, believing it to play a central role in the universe.

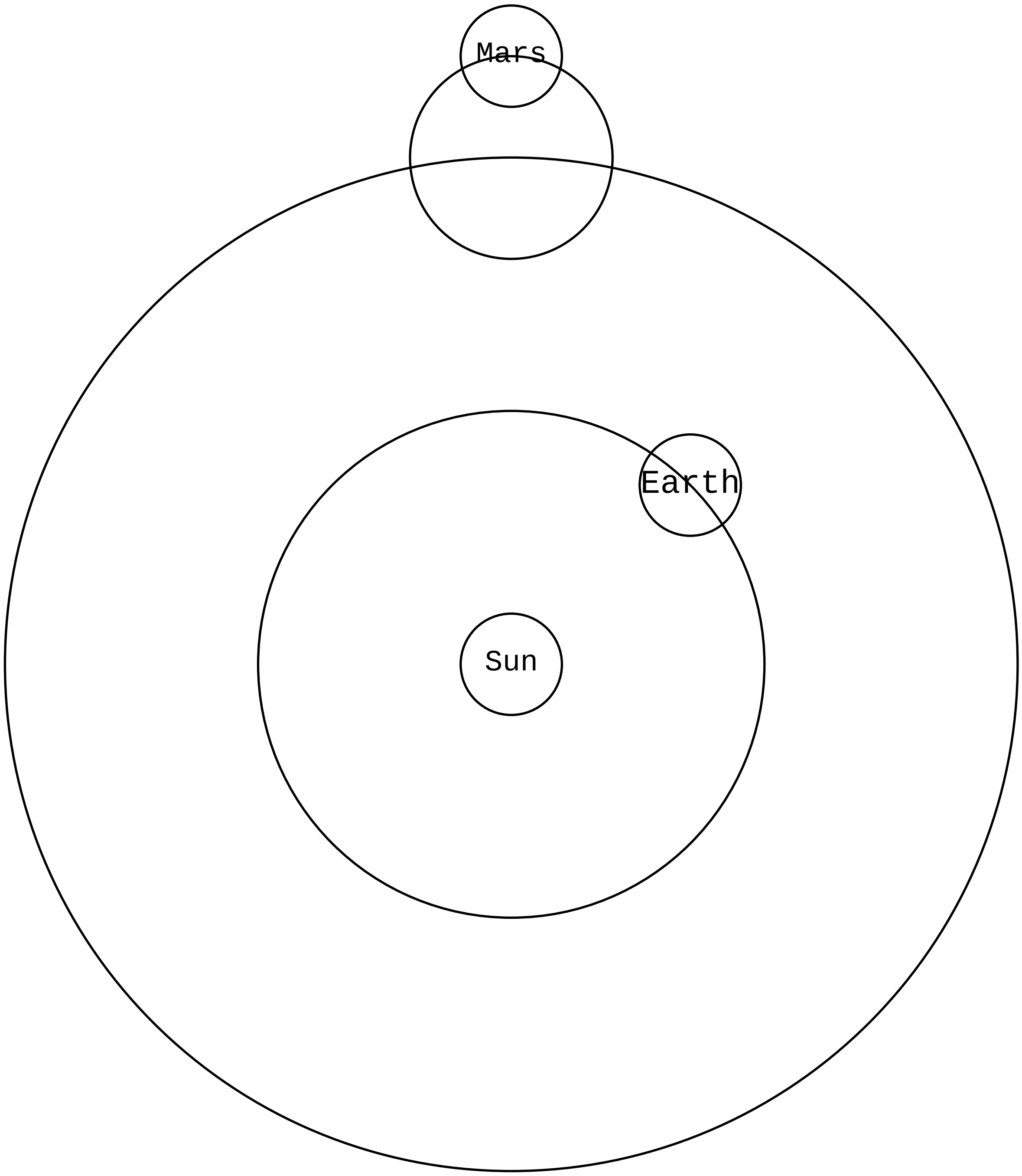
The planets exhibited gross deviations from simple uniform circular motion. The most well known deviation is the retrograde motion of Mars. At certain times, the planet Mars will reverse its course in the Zodiac and actually back up, something that would be impossible if Mars moved in a simple uniform circular orbit. The ancient Greeks resolved this apparent contradiction by introducing the now notorious epicycles. Mars traveled in an epicycle, a circle centered on a point which in turn orbited the Earth. In fact, by Kepler's time, hundreds of epicycles -- epicycles on top of epicycles -- had been introduced to the orbits of Mars and other planets to partially reproduce the observed motion of Mars and the other planets. Even with all of the epicycles, the prevailing theory could not predict the motion of Mars exactly. The predicted location of Mars was often off by a few degrees.

In 1543, well before Kepler's birth, Nicholas Copernicus resurrected the ancient idea that the Sun was the center of the universe in his book *On the Revolutions of Heavenly Spheres*. Most significantly, Copernicus also relied on the ancient notion of uniform circular motion. His precise theory also contained epicycles. It was simpler, with fewer epicycles, but less accurate than the prevailing Ptolemaic theory. There was in fact good reason other than merely religious belief to doubt the new heliocentric theory. Although the numerical data actually supported the Ptolemaic theory, a younger generation that included Galileo and Kepler began to embrace the theory on conceptual grounds.

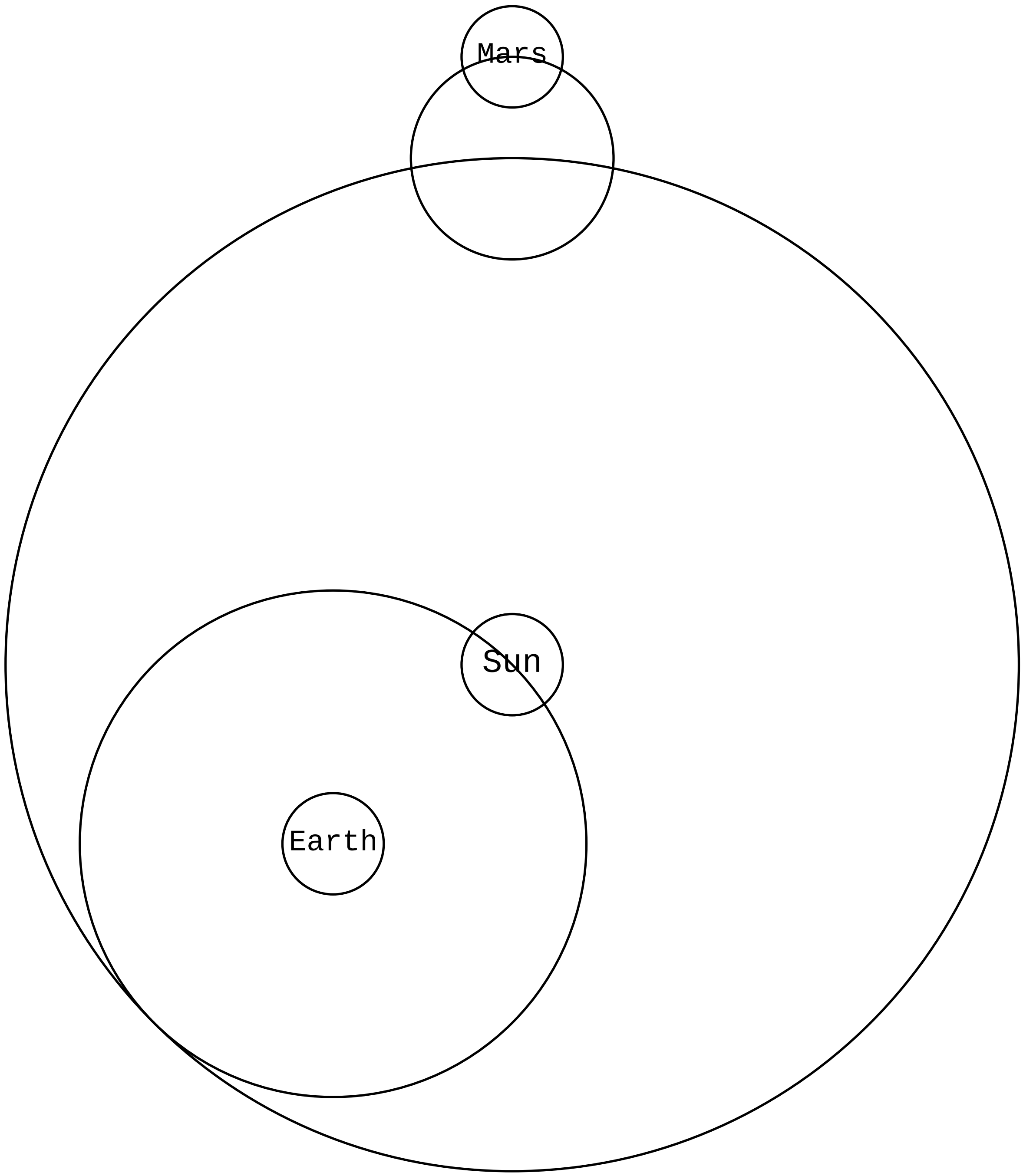
Tycho Brahe introduced a hybrid theory. He envisioned the Sun orbiting the Earth and the other planets orbiting the Sun. In hiring Kepler, he hoped that Kepler could decipher his measurements of the position of Mars and prove his hybrid theory. Although Brahe's theory seems goofy to the modern mind, there were actually some reasons why it seemed possible. On the one hand, the Earth subjectively did not seem to move, whereas the planets did, supporting the common sense notion that the Earth was the stationary center of the solar system. On the other hand, some of the planets such as Mercury and Venus were always found within a certain angle of the Sun and in fact seemed to move around the Sun as viewed from the Earth. One way of resolving these confusing observations was Tycho Brahe's hybrid theory. Tycho Brahe also assumed uniform circular motion and populated his theory with epicycles.


Ptolemaic System
Mars
Sun
Earth

Copernican System



Tycho Brahe’s System



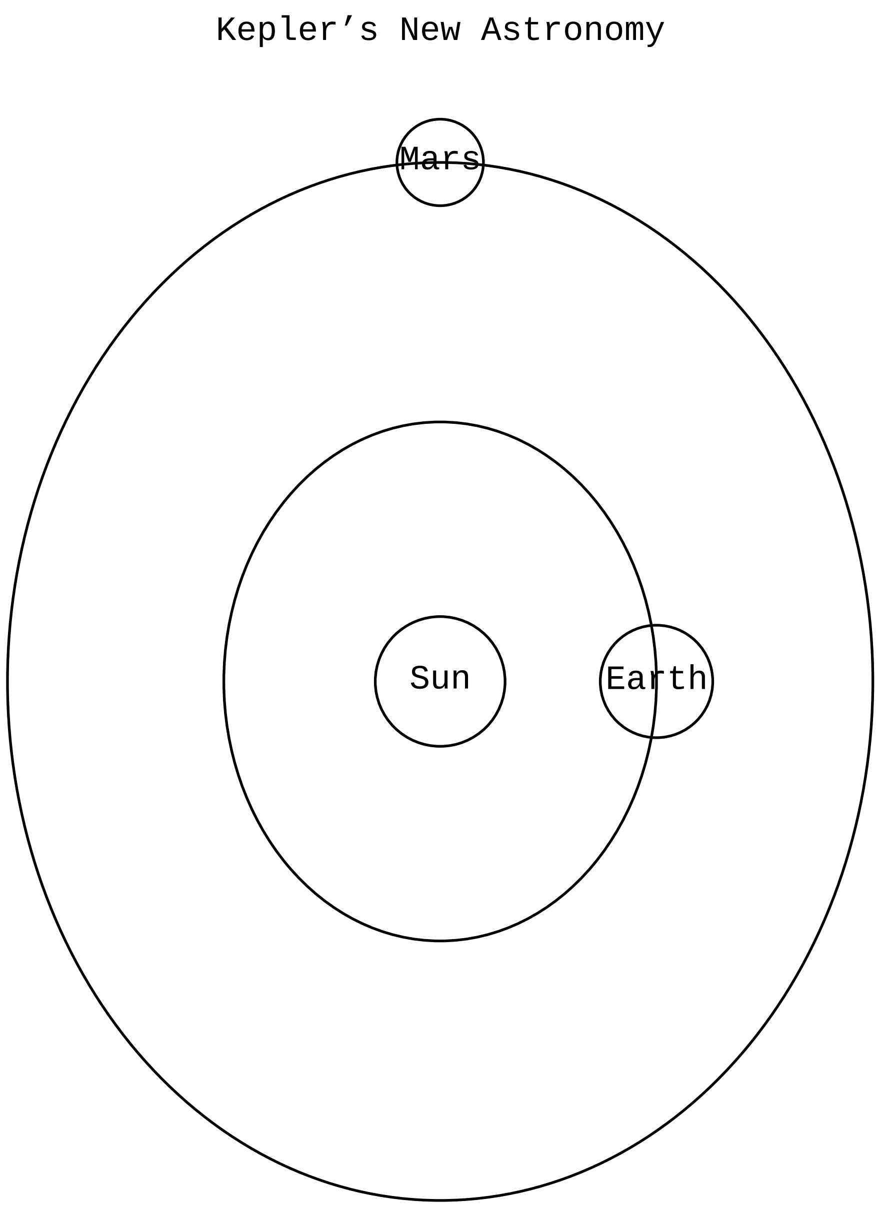
Kepler and the Orbit of Mars

Johannes Kepler was twenty-nine when he arrived in Prague to work for Tycho Brahe. He was very confident. He bet that he could solve the problem of the orbit of Mars in eight days. He lost the bet. Over the next five years, Kepler made hundreds, if not thousands, of calculations using variants of all three theories to explain and predict the motion of Mars. He had access to Tycho Brahe's precision measurements of the position of Mars made using new state of the art instruments. Even with this high quality data, he could not make sense of the orbit of Mars. The Ptolemaic models failed. The Copernican models failed. The Tycho Brahe models failed. He eventually determined that the three approaches were mathematically equivalent. With the right combinations of epicycles, each theory could predict the same observed motion of Mars, which always came out wrong. In fact, the three theories are simply different choices of a coordinate system, a concept that was generally not understood in Kepler's time.

Kepler also engaged in an enormous amount of conceptual analysis of the problem. A believer in the heliocentric Copernican system, he read William Gilbert's book on magnetism and conceptualized the Sun, the Earth, and the other planets as magnets or magnet-like bodies that somehow attracted one another. Gravity, in the modern sense, was unknown at the time. It is likely that these concepts enabled him to make his eventual breakthrough. The analogy to a terrestrial magnet led naturally to the idea that the Sun's influence declined with distance. Thus the planets in an elliptical orbit might travel slower further away from the Sun and faster closer to the Sun. The concept of something like the influence of a magnet radiating outward from the Sun probably made the concept of an elliptical orbit more sensible.

Kepler enjoyed enormous freedom. Rudolf II thought nothing of showering money on hundreds of alchemists, astrologers, and scholars like Kepler. Kepler had received a promotion to Imperial Mathematician shortly before Tycho Brahe's death in 1601, possibly from accidental or intentional mercury poisoning. Tycho Brahe's untimely death meant that Kepler was largely free to pursue his analysis of Brahe's data in any direction without fear that Brahe would be unhappy with disproof of the hybrid theory. Rudolf II did appoint another scholar to supervise Kepler but this scholar seems to have confined himself to brief encouraging comments. Nonetheless, Kepler was unable to figure out what was going on. He was in the process of giving up and writing a book detailing his failed calculations in 1605, when he abruptly, in just a few days, figured out what was going on.

The problem was the assumption of uniform circular motion shared by all three theories. In fact, Mars moved non-uniformly in an elliptical orbit. Once Kepler realized this, he was able to look up the mathematics of the ellipse in the ancient work *Conics* of Apollonius of Perga. This, by the way, is an extremely difficult to read book and it was a substantial undertaking for Kepler to master its contents. In modern terms, the planet Mars swept out equal areas in equal times, traveling faster nearer the Sun and slower farther away. This is a modern formulation of Kepler's so-called "second law", taught in introductory science classes, which is much simpler and clearer than the way that the second law was stated in Kepler's book *Astronomia Nova (New Astronomy)* which was finally published in 1609 announcing his discovery to the world.


Kepler’s New Astronomy
Mars
Sun
Earth

What can we learn from Kepler?

The discovery of the elliptical orbit of Mars shares a number of features in common with many other major scientific discoveries and technological inventions. It took a long time, about five years. Kepler arrived in Prague in January 1600, completing his move to the city from Graz on September 10, 1600, and by his own account solved the problem during the Easter season in 1605. Most major scientific discoveries and technological inventions have taken over five years. In fact, the discovery of the elliptical orbit of Mars is one of the faster scientific discoveries.

The discovery of the elliptical orbit of Mars required a large amount of trial and error. Kepler probably tried somewhere between hundreds and thousands of ideas, none of which worked, during his first five years in Prague. Indeed, the mathematical calculations required only pen and paper and some time, keeping the cost and duration of the trials quite low. This is probably one of the reasons the discovery took so little time compared to other discoveries and inventions.

As with many major scientific discoveries and technological inventions, the discovery involved a long period of frustrating failure, over five years. This is not uncommon. As in many other discoveries (not all), the failures were due to a faulty assumption or deeper systemic problem that probably could not have been recognized from a single failure or a small number of failures. In this case, the assumption that planets moved according to uniform circular motion was so deeply rooted that all of the major competing theories shared this incorrect assumption. It was very difficult for Kepler or anyone to even recognize the existence of the assumption and that it had no solid basis. The lavish support of the Holy Roman Emperor Rudolf II and the death of Tycho Brahe put Kepler in the unusual position of being able to challenge and dismiss the established view with limited opposition.

Experience shows that experience is usually right but not always. Widely accepted knowledge is usually correct but not always. If textbooks say that the Earth is roughly spherical and not flat, they are probably right. It is widely accepted not to jump in front of a speeding car. Nor would one be wise to start jumping in front of speeding cars because one sees one person jump in front of a speeding car and live because the speeding car stops in time to avoid a fatal collision. Now would it be wise to jump in front of a speeding car, even if one saw several cases of people jumping in front of speeding cars and walking away unharmed. Our unwillingness to challenge deeply rooted assumptions is often correct, but not always. It thus correctly requires a large number of failures to call into question a fundamental assumption, a widely accepted "fact". It is probably quite sensible not to challenge a widely accepted assumption until hundreds or thousands of failures have accumulated.

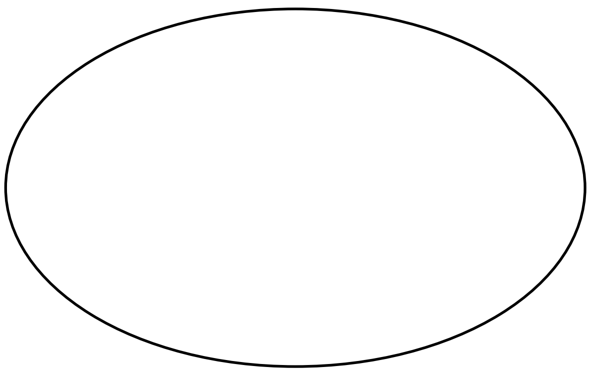
Kepler had a sudden abrupt realization of the answer, a so-called "flash of insight" which took place very rapidly during the Easter season in 1605. This is also a very common feature of many scientific discoveries and technological inventions. Popular accounts of discoveries often emphasize the flash of insight and omit or substantially downplay the large amount of trial and error and frequent long periods of failure that usually precede the flash of insight. This combination tends to create the impression that discoverers and inventors have extreme, almost supernatural, intelligence.

Kepler also engaged in a large amount of conceptual analysis of the problem expressed in words and pictures. He spent a great deal of time thinking about how the Sun might act something like a magnet attracting or influencing the other planets. This conceptual analysis is also very common in major scientific discoveries and inventions, including several other advances in physics. The chemist Michael Faraday arguably worked out Maxwell's equations in words and diagrams of lines of force with negligible mathematical knowledge or training. Indeed, the mathematics that Kepler used was almost entirely Greek geometry expressed through words and diagrams, not modern symbolic algebra, and tables of a trigonometric function known as the *chord* that was much more awkward to use than the modern *sine* and *cosine* functions taught in high school. It was very cumbersome to perform actual calculations of numerical results using these methods. A high school student today can perform many of the calculations that Kepler performed, even without a computer, much more easily using modern trigonometry and symbolic notation. There is no question that modern symbolic algebraic methods make calculations faster and can be programmed easily on a computer. On the other hand, the ancient Greek methods enabled and in fact required explicit visualization of the planetary system, which may have helped Kepler make his breakthrough.

The mathematical formula for an ellipse in modern notation:

$$a\,x^2 + b\,y^2 = c$$

A diagram of an ellipse:



Formula or diagram, which is easier to think about?

The proliferation of epicycles

The story of the epicycles is usually told with a great deal of ridicule and scorn. It is often part of a morality tale contrasting objective rational modern science with medieval religion and superstition. That Copernicus also used epicycles is usually omitted. This is probably unfortunate. The heavy ridicule of the epicycles probably makes it difficult to question or abandon complex mathematical models today. No one today wants to go down in history as the purveyor of epicycles. Ptolemy is known to many as the architect of an "obviously" wrong theory rather than the brilliant scholar that he actually was. In fact, the elliptical orbit of Mars and the other planets was far from obvious, especially given the limited data and mathematical methods of the time.

The epicycles are one of the earliest examples of a problem that bedevils mathematical modeling to this day. No general solution has been found. It is possible to approximate almost any function (such as the measured position of the planet Mars) with a sum, sum of products, or similar composition of many other functions (such as epicycles). With enough building block functions, enough "terms in the expansion" in mathematical jargon, the function can be approximated as accurately as one desires. However, this approximation may fail to predict the values of the function outside of the region in which the approximation was made.

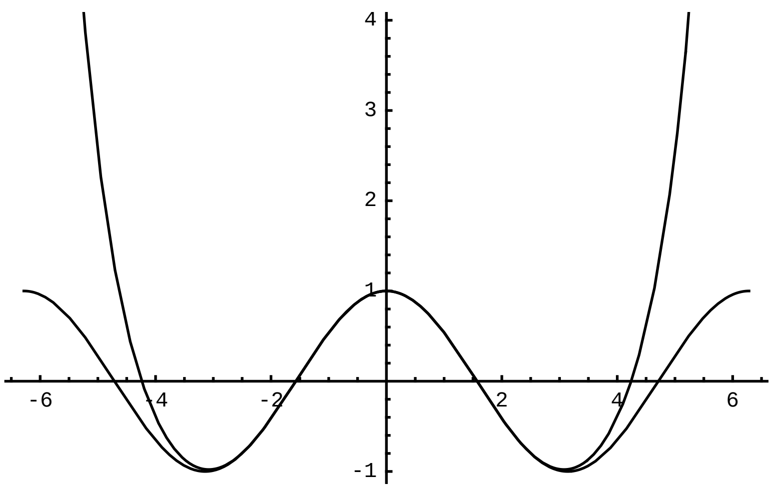
The cosine of basic trigonometry *cos(x)* is a simple periodic function. *cos(wt)* where *w* is a constant angular velocity and *t* is time gives the horizontal or *x* coordinate of a point on the circumference of a wheel of unit radius rotating at a constant speed. The cosine can be approximated by a sum of the powers of *x*. This is shown in both symbolic notation and pictures below. The code below is *Mathematica*, a sophisticated mathematical program widely used in academic research. *Mathematica* is very similar to MATLAB (despite claims to the contrary) which is widely used in industry. For those familiar with computer programming languages, *Mathematica* and MATLAB are scripting languages somewhat similar to Python or Perl with extensive tightly integrated mathematical capabilities. The code is retained to help clarify the illustrations.

```
Series[Cos[x],{x,0,8}]
```

$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}+O[x]^9$$

A power series approximation to the cosine function with five terms (above).

```
Plot[Evaluate[{Cos[x], Normal[Series[Cos[x], {x, 0, 8}]]}], {x, -2 π, 2 π}, ImageSize → 72 × 5]
```



The power series approximation compared to the actual cosine function in the range -6 to 6 units(above). Notice that the approximation diverges widely beyond about four units from the origin of the plot.

```
Plot[Evaluate[{Cos[x], Normal[Series[Cos[x], {x, 0, 8}]]}], {x, -4 π, 4 π}, ImageSize → 72 × 5]
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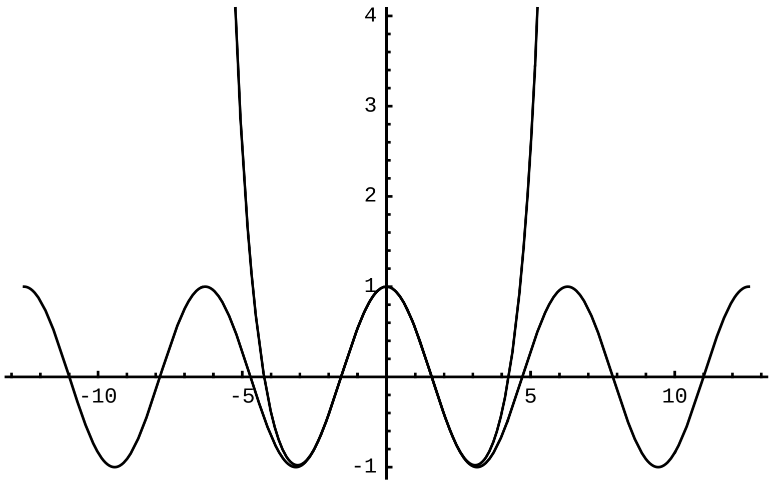


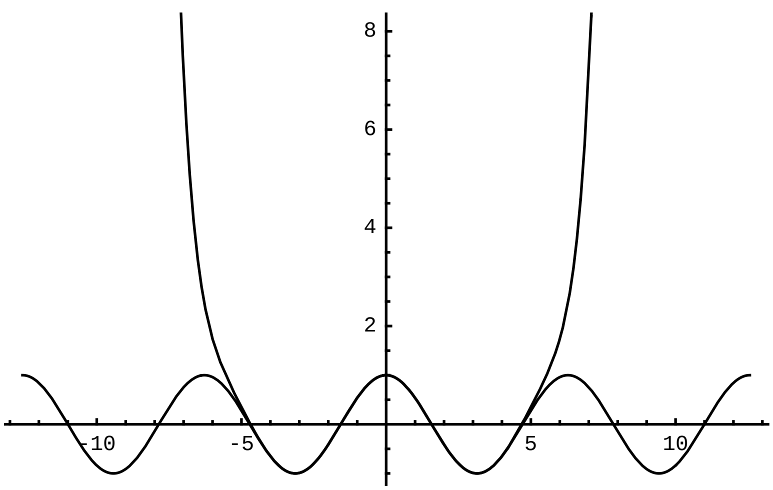
The power series approximation compared to the actual cosine function in the range -10 to 10 units (above). Notice that the approximation diverges widely beyond about four units from the origin of the plot.

Adding more powers of *x* (terms in the expansion) improves the match (below).

```
Series[Cos[x],{x,0,12}]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800} + \frac{x^{12}}{479001600} + O[x]^{13}$$

```
Plot[Evaluate[{Cos[x], Normal[Series[Cos[x], {x, 0, 12}]]}], {x, -4 π, 4 π}, ImageSize → 72 × 5]
```

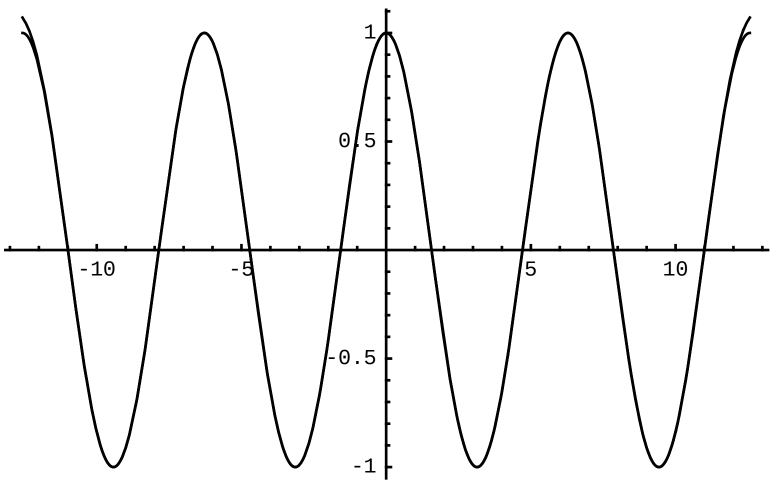


Adding even more powers of *x* improves the match even more (below).

```
Series[Cos[x],{x,0,32}]
```

$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800}+\frac{x^{12}}{479001600}-\frac{x^{14}}{87178291200}+\frac{x^{16}}{20922789888000}-\frac{x^{18}}{6402373705728000}+\frac{x^{20}}{2432902008176640000}-\frac{x^{22}}{1124000727777607680000}+\frac{x^{24}}{620448401733239439360000}-\frac{x^{26}}{403291461126605635584000000}+\frac{x^{28}}{304888344611713860501504000000}-\frac{x^{30}}{265252859812191058636308480000000}+\frac{x^{32}}{263130836933693530167218012160000000}+O[x]^{33}$$

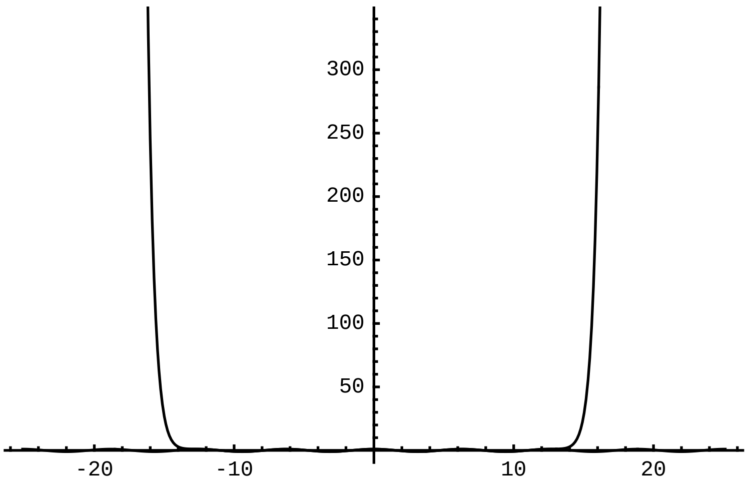
```
Plot[Evaluate[{Cos[x], Normal[Series[Cos[x], {x, 0, 32}]]}], {x, -4 π, 4 π}, ImageSize → 72 × 5]
```



The approximation is now almost exact from -10 to 10 units (above).

But the approximation still fails when one moves far enough away from the region where the approximation was made (below):

```
Plot[Evaluate[{Cos[x], Normal[Series[Cos[x], {x, 0, 32}]]}], {x, -8 π, 8 π}, ImageSize → 72 × 5]
```



For illustrative purposes I have intentionally chosen a set of building block functions, the simple powers of *x*, that do not share the periodic property of the function being approximated (the cosine). These approximations fail very quickly outside of the region where the approximation is made. The simple powers of *x* blow up as *x* gets large, whereas periodic functions such as *cos(x)* oscillate within a fixed range. The measured positions of Mars and the other planets were loosely periodic functions. The ancients sensibly tried to model the motion of the planets with the one periodic function that they understood very well, the uniform rotation of a wheel. When the building block functions share some or all of the gross features of the function being approximated, the failure of the approximation as one moves outside of the region used to make the approximation is often less dramatic. The approximation may have some predictive power. This was the case with the models of planetary motion that used epicycles.

Astronomy and physics have progressed enormously since Kepler's time. Recent years have seen many space missions launched seeking to explore the frontiers of astronomy and physics. Probably the most well known is the Hubble Space Telescope which discovered anomalous redshifts leading to the postulation of the mysterious dark energy. Other space-borne astronomy and physics missions include the Kepler mission to detect extra-solar planets including Earth-sized planets, Gravity Probe B which is a high precision test of Einstein's General Theory of Relativity, gamma ray detectors such as the Fermi Gamma-ray Space Telescope (formerly known as GLAST) and PAMELA, and a number of others. Many of these missions depend on sophisticated mathematical modeling to analyze their data. The Hubble redshift data must be compared to complex models of the Big Bang combining General Relativity and various particle physics theories of the early universe. The Kepler mission seeks to detect extra-solar planets by detecting the slight decrease in the light from stars as the planet crosses in front of the star. This requires detailed models of the stars, the light propagation, the observing instrument on board the Kepler spacecraft to distinguish the planetary transit from possible backgrounds such as fluctuations in the star's light output. On Earth, particle accelerators and other ground-based experiments that seek to directly detect the hypothetical weakly interacting particles, modeled loosely on the neutrino, that might comprise the dark matter must also construct complex mathematical models to analyze their sometimes confusing and baffling data.

With modern mathematical notations, computers, and sophisticated computer software such as *Mathematica* and MATLAB, scientists and engineers today can create far more complex models than the Ptolemaic models of Kepler's time. In Kepler's time it was an arduous process to compute the predictions of the models by hand calculation. Kepler disliked tedious calculation and preferred to work with concepts. This probably motivated him to find a simpler model and also to spend a great deal of time thinking about how the Sun might influence or control the orbiting planets. In astronomy and physics, extremely complex models have proliferated.

Dark Matter

The standard Newtonian theory of gravity predicts that orbiting objects travel with decreasing angular velocity the further away they are from the object that they orbit. Thus, Mercury, which is the closest planet to the Sun, takes only 81 days to orbit the Sun. The Earth takes one year. Mars takes about two years. Jupiter takes about twelve years. Saturn takes about thirty years. Pluto takes about two centuries. The duration of the planetary orbits is in fact governed by Kepler's so-called "third law" which he discovered in 1619 when he was about forty-eight, after almost twenty years of studying Tycho Brahe's data. This is because the force of gravity is proportional to the inverse of the square of the distance from the object, for example the Sun in the solar system. However if the solar system was filled with mysterious invisible matter, the force of gravity would not be proportional to the inverse of the square of the distance from the Sun because the effective mass of the Sun would increase with the distance from the Sun. In the Newtonian theory of gravity, all of the invisible matter inside the orbit of the planet would appear as if concentrated at the center of the orbit, the Sun's location.

Similarly stars in the arms of our galaxy, the Milky Way, should orbit the galactic core with lower angular velocity (longer orbit durations) the farther out they are from the galactic core. The Milky Way is a spiral galaxies with several arms like pinwheel. The Sun is located about two thirds of the way out on one of the arms from the galactic core. The galactic core is very dense, highly energetic, and may contain a black hole. Since the 1930's, it has been known that the angular velocity of the stars in the Milky Way drops off much less rapidly than expected according to Newtonian gravity, assuming that the visible matter, the stars, is the only matter in the galaxy. The anomalous angular velocity distribution of the stars in the Milky Way was first noticed by the astronomer Fritz Zwicky at Caltech in the 1930's. If the Milky Way galaxy is filled with "dark matter" that cannot be seen out to the radius of the galactic arms, then one can explain the observed angular velocity distribution of the stars.

Since the 1930's astronomers have measured the orbits of galaxies in clusters of galaxies, clusters of galaxies in clusters of clusters, and so forth. They have found similar anomalies in the angular velocity of galaxies at these larger scales. Again the anomalous high angular velocities of the galaxies and clusters of galaxies may be explained by postulating a mysterious dark matter that fills the universe. It is doubtful that one can explain the anomalous angular velocities at different scales by the same type of dark matter. Thus, the dark matter responsible for the orbits of the stars in the Milky Way is probably different from the dark matter responsible for the orbit of the Milky Way within the local super-cluster of galaxies.

Dark Energy

Albert Einstein discovered the General Theory of Relativity, a theory of gravity that reconciled gravity with the Special Theory of Relativity. In 1919 the astrophysicist Arthur Stanley Eddington measured the deviation of star light passing close to the Sun during a solar eclipse to match the value predicted by Einstein's theory, which differed from the value predicted by Newton's theory of gravitation. Eddington announced this discovery to the war weary world at a special meeting of the British Royal Society on November 6, 1919 in what appears to have been a carefully staged media event. Einstein became world-famous, the subject of front page headlines, on November 7, 1919, curiously the second anniversary of the Bolshevik Revolution. There are very serious doubts about the accuracy of Eddington's 1919 measurements although subsequent less well known measurements of solar eclipses have reportedly confirmed Einstein's theory. In Eddington and Einstein's time, General Relativity was largely a matter of academic interest, although even then there were speculations about space warps and interstellar travel. Today the Global Positioning System (GPS) successfully incorporates small effects from General Relativity to compute precise time and position estimates.

Einstein tried to apply the General Theory of Relativity to cosmology and the origin and evolution of the universe. At the time, the prevailing belief was that the universe was static, of infinite age, neither expanding nor contracting. Unfortunately, the equations of General Relativity predicted a dynamic universe, either expanding or contracting, with a beginning and possibly an end. This has come to be known as the Big Bang in which the universe expands from a point to the universe we know. If the force of the Big Bang is large enough, the universe will continue to expand for all eternity. If the force of the Big Bang is too small, the universe will collapse back into a point. Einstein was not particularly happy with this result and he added a fudge factor known as the cosmological constant to the General Theory of Relativity to get the static universe that he expected. The cosmological constant represented a mysterious force that kept the universe from collapsing.

In 1929 Edwin Hubble at the Mount Wilson Observatory above Pasadena, California published an analysis of observations of galaxies outside of the Milky Way. It had been known for some time that "extra-galactic nebulae" as they were called at the time exhibited significant red shifts. A red shift means that the spectrum of known elements such as hydrogen was shifted toward the red end of the spectrum. This can happen if the galaxies are receding from us. Even more remarkably, the red shift was often larger the farther away the galaxy. If one interpreted the red shifts as meaning that a galaxy was receding from us, Hubble found a linear relationship between the velocity of recession of the galaxy and its distance from the Earth. Hubble's so-called constant is now measured to be about 72.4 kilometers per second per million parsecs (megaparsec). A parsec is a unit of distance equal to 3.2615 light years. A light year is the distance traversed by light in one year. What this means is that a galaxy one megaparsec away will have a redshift consistent with a velocity of recession of 72.4 kilometers per second. A galaxy one hundred (100) megaparsecs away will have velocity of recession of 7240 kilometers per second. A galaxy one thousand megaparsecs away will have a redshift consistent with a velocity of recession of 72,400 kilometers per second. The speed of light is about 300,000 kilometers per second. Some distant, presumably ancient objects, such as quasars have redshifts consistent with a velocity of recession close to the speed of light.

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Plot[ 72.4 x, {x, 0, 5000}, PlotLabel->"Hubble's Velocity versus Distance Relationship", Imag
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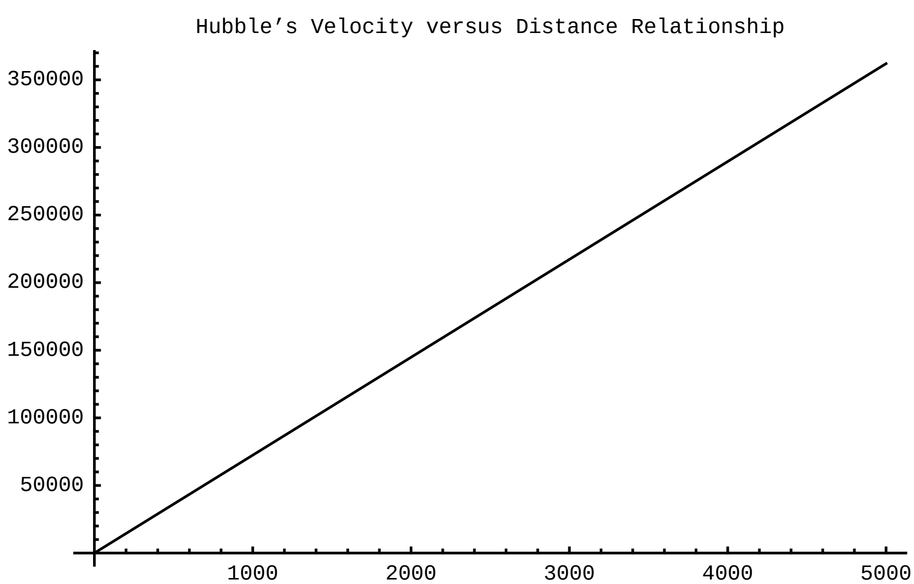

It is worth mentioning that Hubble's initial paper in the Proceedings of the National Academy of Sciences was an analysis of only twenty-four "extra-galactic nebulae" that Hubble deemed to have well measured distances. Hubble initially estimated his constant as 500 kilometers per second +/- about 50 kilometers per second per million parsecs (megaparsecs). This is quite different from the modern value of 72.4 kilometers per second per million parsecs. The determination of Hubble's constant has a long and contentious history. Hubble estimated the constant at 526 kilometers/second/megaparsec in 1936 based on more comprehensive measurements. Walter Baade estimated the constant at 200 kilometers/second/megaparsec in 1950. Allan Sandage estimated the constant in the range 50-100 kilometers/second/megaparsec in 1958. Racine and Sandage estimated a value of 77 kilometers/second/megaparsec in 1965. Abell and Eastman estimated a value of 53 kilometers/second/megaparsec in 1968. Van den Bergh estimated a value of 95 kilometers/second/megaparsec in 1968. Allan Sandage and Gustav Tammann estimated a value of 55 kilometers/second/megaparsec in 1975 and so on. It is now thought that the galaxies are much farther away than Hubble thought, hence much of the difference. It is also worth noting that Hubble did not discover the extra-galactic redshifts as is often stated in many sources. The redshifts had been known for some time.

Why would all of the galaxies in the universe be running away from our galaxy? Why would the velocity increase proportional to the distance from our galaxy? Hubble's results were consistent with the predictions of the General Theory of Relativity without the cosmological constant. The physicist Willem De Sitter and others had worked out the implications of General Relativity without the cosmological constant, the nucleus of the modern Big Bang theory, during the 1920's. Hubble was well aware of these predictions, citing De Sitter's work in his 1929 analysis of the redshifts. Einstein is frequently quoted as calling the cosmological constant his "greatest blunder".

The conventional Big Bang theory predicts that the expansion of the universe should be slowing down over time due to the gravitational attraction of the matter in the universe. What this means is that Hubble's velocity versus distance relationship should not be a perfect straight line. There should be a deviation as one looks very far away, very far back in time. The expansion should be decelerating.

One of the missions of the Hubble Space Telescope is to precisely measure the Hubble constant and its rate of deceleration, a hotly debated topic in astronomy. These measurements unexpectedly showed that the relationship between the redshift and the distance was not what was predicted by the General Theory of Relativity. The expansion of the universe appears to be accelerating. One can salvage the General Theory of Relativity by restoring the cosmological constant with an appropriate value, not sufficient to produce a static universe but strong enough to accelerate the expansion of the universe. Einstein's "greatest blunder" is now being hailed as yet another brilliant prediction by Einstein. Heads, Einstein is a genius. Tails, Einstein is a genius. Astronomers and physicists have postulated a mysterious dark energy that permeates the universe to explain the anomalous redshift observations and to justify the non-zero cosmological constant.

Particle Physics and Dark Stuff

The most popular explanations for the various kinds of dark matter and the mysterious dark energy are various hypothetical subatomic particles. Dark matter and dark energy lie at the growing convergence of astronomy and particle physics, so-called "astro-particle" physics. Particle physicists like to say that they are recreating the conditions a fraction of a second after the Big Bang in their particle accelerators.

Particle physicists observe a number of subatomic particles that behave very much like our common sense notion of particles. These include the electron, the muon, the pion, the kaon, the proton, and a few others. These are almost directly visible in particle physics detectors. These particles leave behind trails of energy, so-called tracks. In the old days of cloud chambers and bubble chambers, these tracks were photographed and studied by eye. Today, tracking detectors are often chambers filled with gases such as carbon dioxide or large arrays of silicon detectors similar or identical to the CCD's in digital cameras. The particles leave trails of ionization in the gas or silicon and then can be measured electronically. The tracks can be reconstructed in detail and displayed on computers.

Some particles such as the D mesons and the B mesons are very short lived, about one trillionth of a second (a picosecond). Nonetheless, they live long enough to decay a measurable distance from where they are created, a few hundred microns. A micron is about the breadth of a human hair. These mesons decay to the longer lived particles such as electrons and pions that leave clear tracks in the detector. In particle colliders, beams of electrons and positrons or protons and protons (sometimes anti-protons) collide at a precise point at the center of a giant particle detector. The very center of the detector contains a high precision silicon tracking device, a so-called vertex detector. It is possible to see the displaced vertex made by the decay products of a D meson, a B meson, and some other particles. There are a few dozen particles that produce displaced vertices.

There are thousands of particles, sometimes known as nuclear resonances, that are supposedly extremely short lived, about one trillionth of a trillionth of a second. They produce neither identifiable tracks in a detector nor displaced vertices. They appear only as peaks or shoulders in reconstructed energy and momentum distributions. The rho meson is one of the first nuclear resonances to be discovered. The Particle Data Group's Particle Data Book contains thousands of these "particles". One is now some distance from the common sense notion of a particle.

Some additional particles known only through indirect evidence are the quarks, thought to be the building blocks of the proton, neutron, and other strongly interacting particles. Quarks were "detected" in electron scattering experiments in which high energy electron beams were scattered off of targets, notably liquid hydrogen. Sometimes the electrons would bounce off at very sharp angles such as ninety degrees, suggesting the presence of small charged particles inside the proton, the hydrogen nucleus. Quarks were also inferred by Murray Gell-Mann and George Zweig from the energy spectrum of the strongly interacting sub-atomic particles, including many of the nuclear resonances. In the 1970's and early 1980's, there were many attempts to detect isolated quarks. But quarks have never appeared as tracks in a particle detector. Particle physicists developed a theory that the binding force between the quarks actually grew indefinitely with distance. Inside the tiny proton, the quarks acted very much like free particles. Attempting to pull the quarks apart would cause the binding force to grow without limit. Thus free quarks could never be observed. This theory recently received a Nobel Prize.

A peculiar particle that appears well established is the neutrino. The neutrino was first postulated to explain radioactive decays of certain isotopes in which energy seemed to disappear. The laws of conservation of energy and momentum seemed to be violated. The theory was that a particle that could not be detected or interacted very weakly with ordinary matter carried off the missing energy and momentum. This particle, the neutrino, was eventually detected in experiments near nuclear reactors. Nuclear reactors theoretically produce vast numbers of neutrinos when in operation. A neutrino detector is positioned outside the reactor. The reactor shielding should absorb nearly all known radiation: neutrons, electrons, etc. In fact, when the nuclear reactor is turned on, energy will appear in the detector as a few of the neutrinos interact with the detector material. Neutrinos interact so weakly with ordinary matter that they can pass through the entire Earth.

Over the years particle physicists have postulated a wide range of particles: the Higgs particle, supersymmetric particles, "glueballs", and others. The dark matter may be particles somewhat like the neutrino, but far more massive, sometimes known as WIMP's or Weakly Interacting Massive Particles. The dark energy could also be produced by various hypothetical subatomic particles. Several space missions are already in orbit to detect gamma rays from dark matter or dark energy particles. More are planned or proposed. In addition, particle physicists are constructing ground-based and underground detectors to directly detect the weakly interacting particles that may comprise dark matter. These detectors are similar to existing underground neutrino detectors (formerly proton decay experiments) such as Kamiokande in Japan.

One can postulate mysterious subatomic particles to explain almost any phenomenon. The hypothetical particles appear as certain functions such as the Cauchy-Lorentz (Breit-Wigner) function in mathematical models that seek to make precise predictions. Just as with epicycles, one can approximate any function with a large enough sum of Cauchy-Lorentz (Breit-Wigner) functions. Are dark matter and dark energy today's epicycles?

Mathematical modeling taken to an extreme?

The use of computers and complex mathematical models has proliferated in many fields today besides astronomy and physics, most notoriously in finance. There have been many attempts to use complex mathematical models in artificial intelligence and pattern recognition. Today, artificial neural networks, which attempt to emulate human intelligence and pattern recognition, are complex sums and products of simple functions. They fail at a several percent level which is usually unacceptable for practical applications. State of the art speech recognition engines have around 150,000 fitted parameters. Speech recognition engines also fail at a several percent level. In space, more accurate artificial neural networks could enable probes such as *Spirit* and *Opportunity* on Mars to automatically avoid obstacles and immediately respond to threats without waiting many minutes for commands from Earth. Effective speech recognition would enable astronauts in cumbersome space suits to easily and safely operate a wide range of automated equipment by voice command.

Not only do modern mathematical notations, computers, and sophisticated computer software make it possible to construct and use models far more complex than the Ptolemaic system, they also make it possible to generate and try endless variations on theories on a scale that Kepler probably could not have imagined. The superstrings theory that dominates modern theoretical physics has evolved into a bewildering "landscape" of endless variants of the theory. In principle, this ability to try many variations of a theory quickly, something that took Kepler five years with three different major theories, should accelerate the rate of progress. This however requires the willingness to discard a favored idea as Kepler did and the freedom to do so, which Kepler enjoyed. The main physics preprint server *arxiv.org* is flooded with vast numbers of technically sophisticated theoretical papers seeking to explain dark matter, dark energy, and numerous fleeting anomalous observations from both space-borne experiments and particle accelerators, yet actual progress seems minimal. Kepler's experience suggests that the problem may be too many trials and not enough thinking, the conceptual analysis that guided Kepler to his final answer.

Conclusion

Kepler's experience suggests that it is extremely hard to find simple highly predictive mathematical expressions to explain seemingly complex or confusing data. It takes both a lot of trial and error, a large amount of conceptual analysis, and the flexibility and freedom to discard a cherished idea. The answer may seem "obvious" after the fact but this is an illusion caused by knowing the right answer. Kepler was fortunate that the mathematics of the ellipse had already been worked out and that the ellipse described the motions of Mars well. If the orbit of Mars had been governed by another function not known to the mathematics of his time, the problem would have been much harder to solve.

In mechanical invention, the cost and duration of trials can be an important factor in the likelihood of success. If the cost and duration of the trial is too high relative to the available budget and time, too few trials can be performed and failure is almost certain. It usually takes hundreds to thousands, or even more, trials to make a major invention or discovery. In contrast, this has always been much less of a problem in mathematics, theoretical physics, data analysis and similar fields. Even in Kepler's time with tedious hand calculations the cost and duration of a trial was small. Modern notations, computers, and sophisticated software have greatly reduced this burden. One of the advantages of tedious hand calculation is that it encourages careful thought about what to calculate. Modern notations, computers, and software make it much easier to both construct extremely complex models and just try all sorts of ideas without careful conceptual analysis. The example of Kepler suggests more reliance on conceptual analysis and less reliance on computers and symbolic manipulation.

The orbital dynamics used in successful space missions, the Global Positioning System, and other modern applications owe their existence to Kepler's discovery. Similar successes in mathematical modeling today may reveal hidden connections between fundamental forces such as gravity and electromagnetism (even anti-gravity or warp drives might result), reproduce useful aspects of human intelligence on computers, and solve other outstanding problems.

About the Author

John F. McGowan, Ph.D. *is a software developer, research scientist, and consultant. He works primarily in the area of complex algorithms that embody advanced mathematical and logical concepts, including speech recognition and video compression technologies. He has extensive experience developing software in C, C++, Visual Basic, Mathematica, and many other programming languages. He is probably best known for his AVI Overview, an Internet FAQ (Frequently Asked Questions) on the Microsoft AVI (Audio Video Interleave) file format. He has worked as a contractor at NASA Ames Research Center involved in the research and development of image and video processing algorithms and technology. He has published articles on the origin and evolution of life, the exploration of Mars (anticipating the discovery of methane on Mars), and cheap access to space. He has a Ph.D. in physics from the University of Illinois at Urbana-Champaign and a B.S. in physics from the California Institute of Technology (Caltech). He can be reached at* jmcgowan11@earthink.net

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