From Kepler’s Echinus to Rubik’s Stellated Dodecahedron

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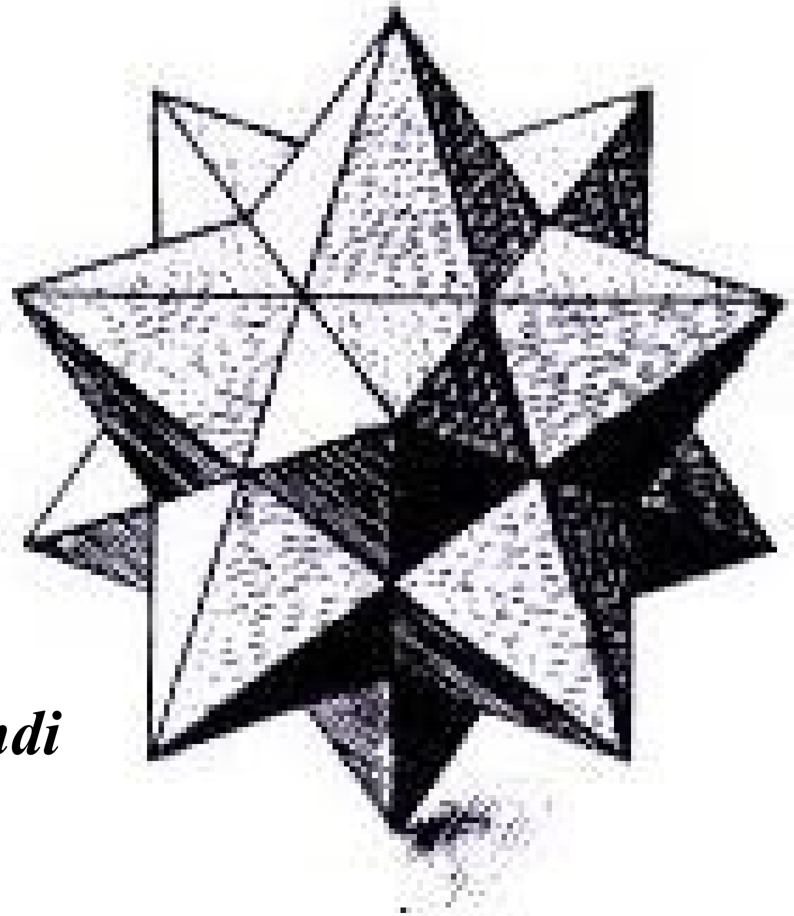
Kepler's Echinus



Harmonice Mundi
1619

small stellated dodecahedron

Echinus = hedgehog

http://bestiary.ca/beasts/beastgallery217.htm#
Bestiarius - Bestiary of Anne Walsh
Kongelige Bibliotek (National Library of Denmark)
http://bestiary.ca/imagesources/imgsrc1629.htm

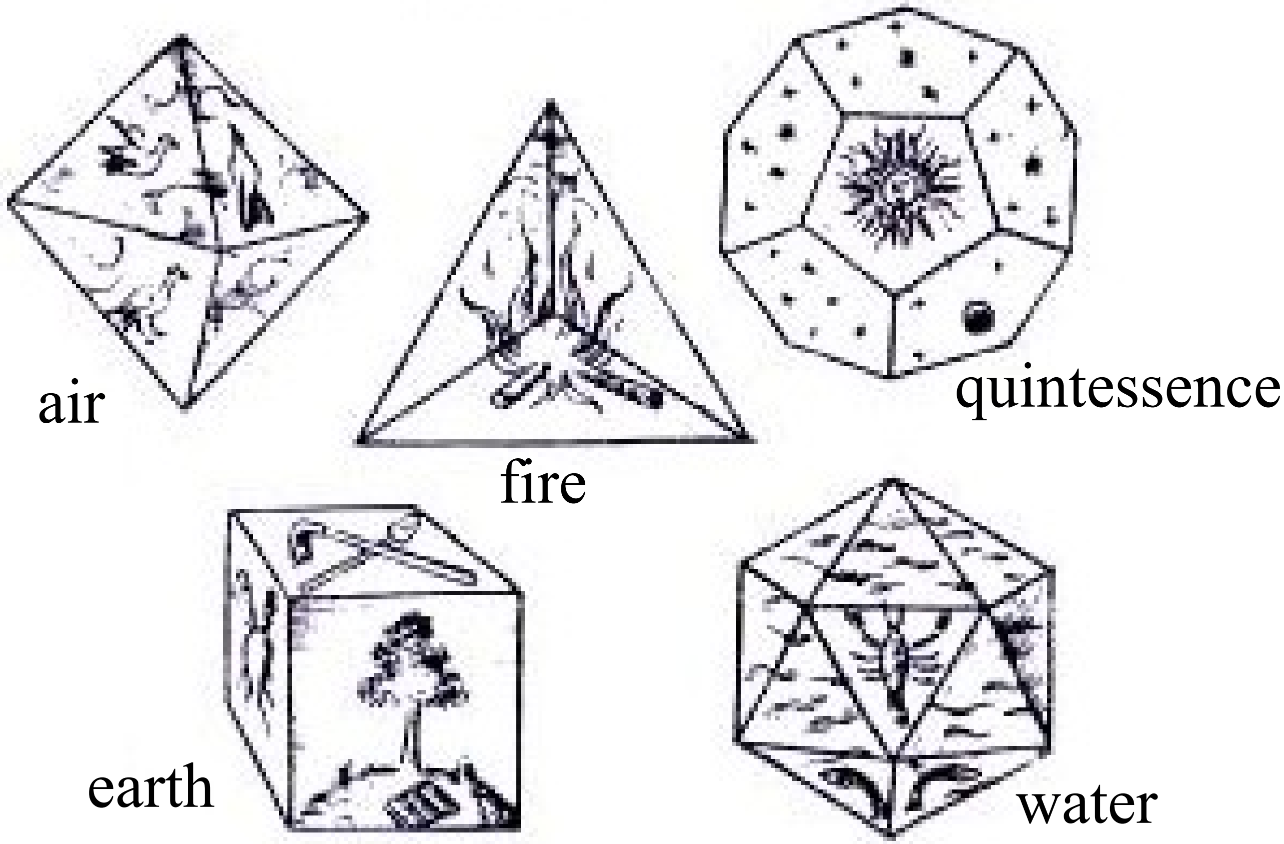
Today's echinus

Edible sea urchin
Echinus esculentus

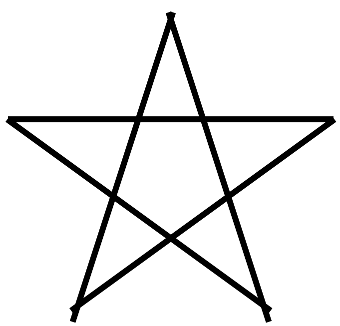
20cm

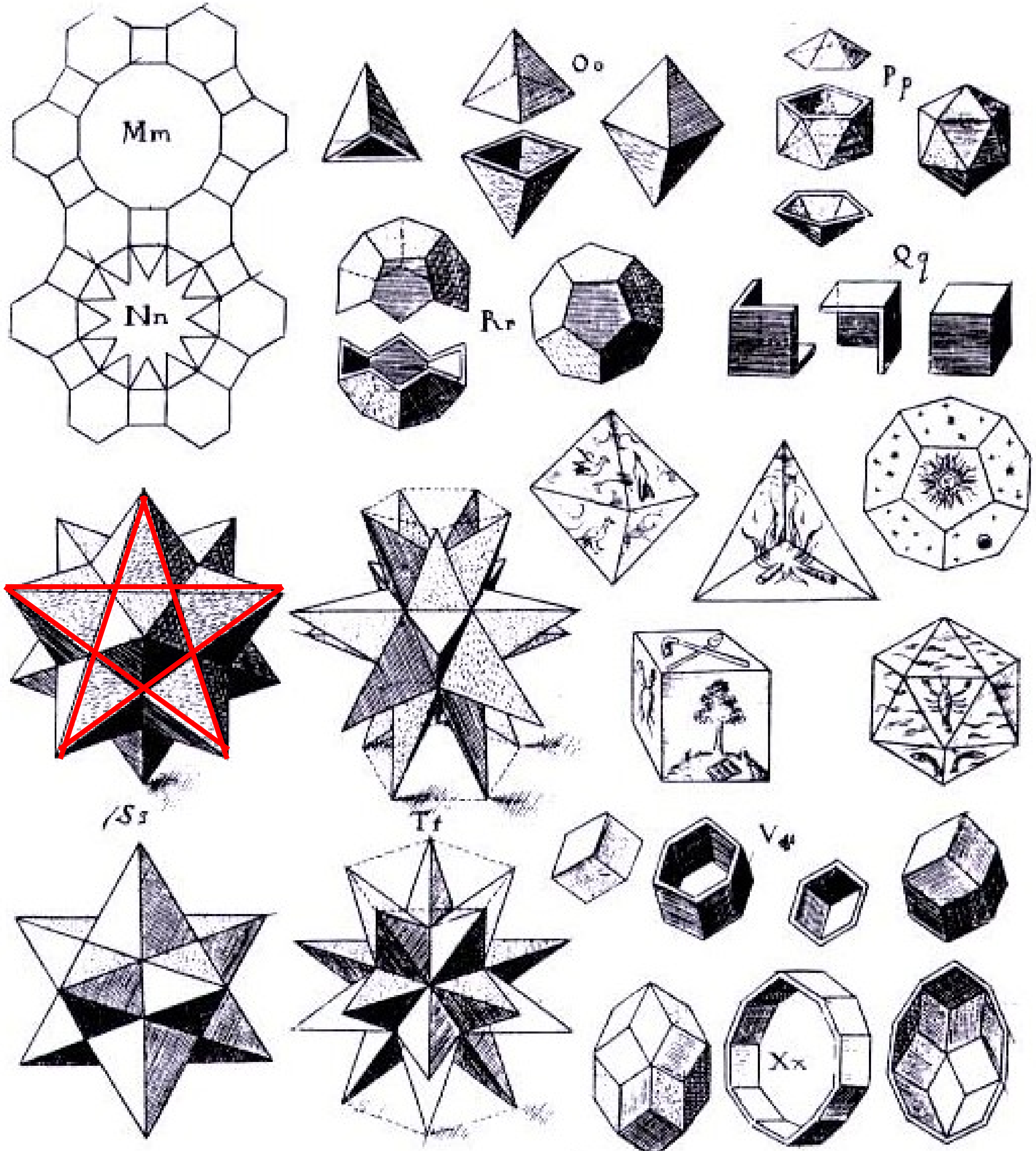
www.marlin.ac.uk/species/Echinusesculentus.htm

Platonic polyhedra

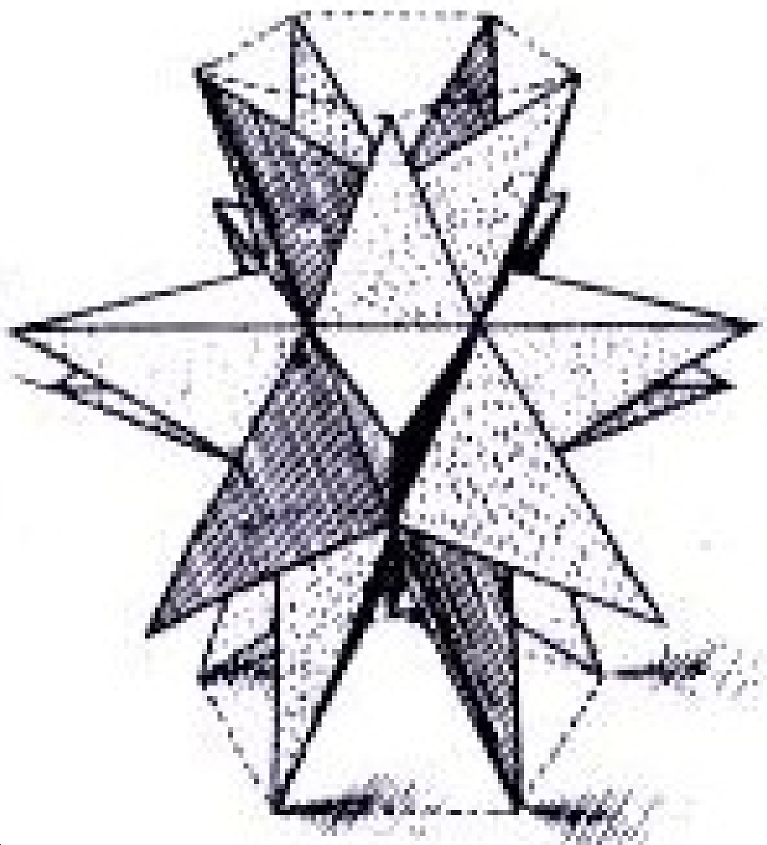


What's regular?





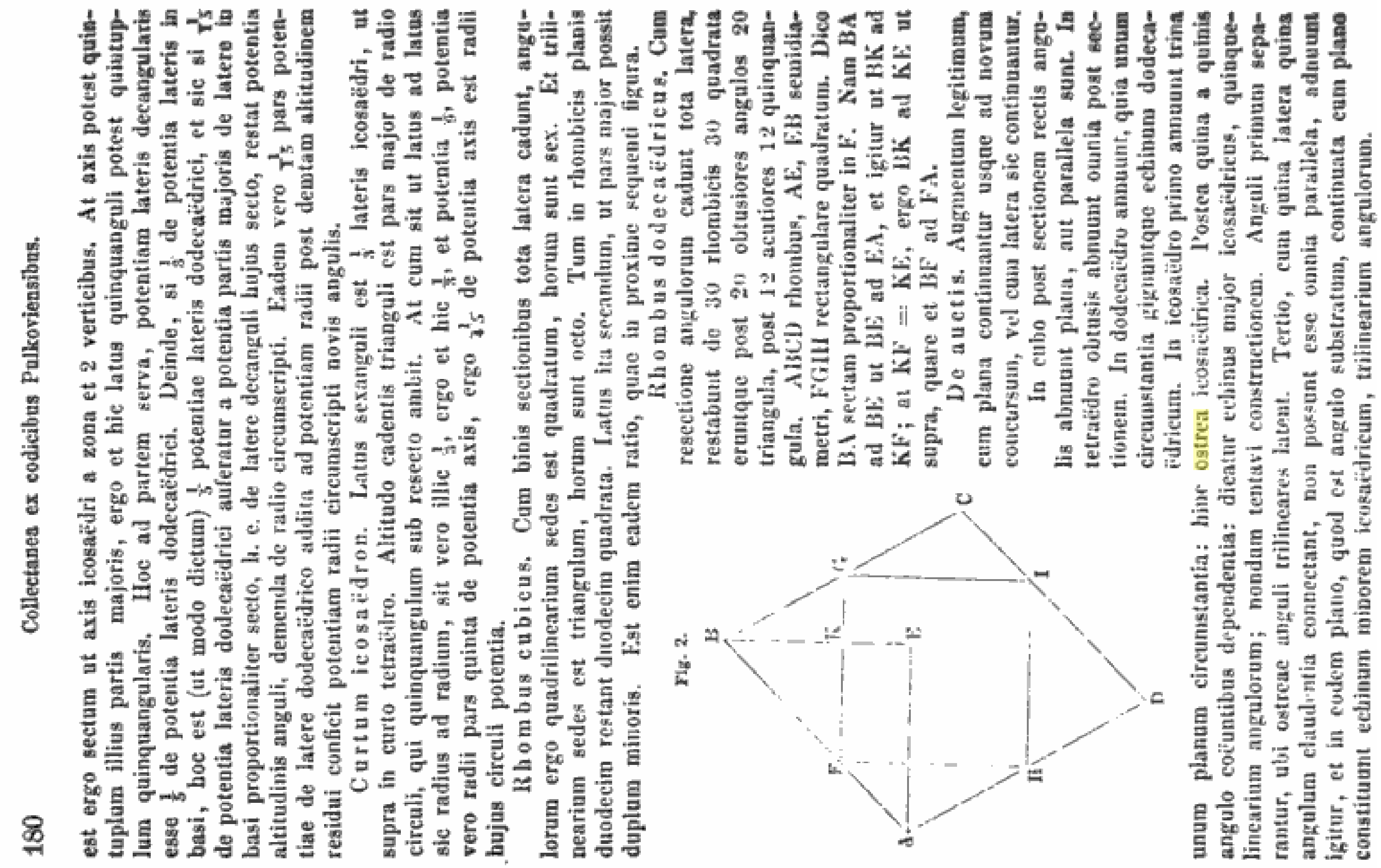
Kepler's Ostrea



(Jamnitzer 1568)

great stellated dodecahedron

Why oyster?



est ergo sectum ut axis icosaëdri a zona et 2 verticibus. At axis potest quintuplum illius partis majoris, ergo et hic latus quinquanguli potest quintuplum quinquangularis. Hoc ad partem serva, potentiam lateris decangularis esse $\frac{1}{5}$ de potentia lateris dodecaëdrici. Deinde, si $\frac{1}{3}$ de potentia lateris in basi, hoc est (ut modo dictum) $\frac{1}{5}$ potentiae lateris dodecaëdrici, et sic si $\frac{1}{15}$ de potentia lateris dodecaëdrici auferatur a potentia partis majoris de latere in basi proportionaliter secto, h. e. de latere decanguli hujus secto, restat potentia altitudinis anguli, demenda de radio circumscripti. Eadem vero $\frac{1}{15}$ pars potentiae de latere dodecaëdrico addita ad potentiam radii post demtam altitudinem residui conficit potentiam radii circumscripti novis angulis.

Curtum icosaëdron. Latus sexanguli est $\frac{1}{3}$ lateris icosaëdri, ut supra in curto tetraëdro. Altitudo cadentis trianguli est pars major de radio circuli, qui quinquangulum sub resecto ambit. At cum sit ut latus ad latus sic radius ad radium, sit vero illic $\frac{1}{3}$, ergo et hic $\frac{1}{3}$, et potentia $\frac{1}{9}$, potentia vero radii pars quinta de potentia axis, ergo $\frac{1}{45}$ de potentia axis est radii hujus circuli potentia.

Rhombus cubicus. Cum binis sectionibus tota latera cadunt, angulorum ergo quadrilinearium sedes est quadratum, horum sunt sex. Et trilinearium sedes est triangulum, horum sunt octo. Tum in rhombicis planis duodecim restant duodecim quadrata. Latus ita secandum, ut pars major possit duplum minoris. Est enim eadem ratio, quae in proxime sequenti figura.

Fig. 2.

Rhombus dodecaëdricus. Cum resectione angulorum cadunt tota latera, restabunt de 30 rhombicis 30 quadrata eruntque post 20 obtusiores angulos 20 triangula, post 12 acutiores 12 quinquangula. ABCD rhombus, AE, EB semidiametri, FGHI rectangulare quadratum. Dico BA sectam proportionaliter in F. Nam BA ad BE ut BE ad EA, et igitur ut BK ad KF; at KF = KE, ergo BK ad KE ut supra, quare et BF ad FA.

De auctis. Augmentum legitimum, cum plana continuantur usque ad novum concursum, vel cum latera sic continuantur.

In cubo post sectionem rectis angulis abnuunt plana, aut parallela sunt. In tetraëdro obtusis abnuunt omnia post sectionem. In dodecaëdro annuunt, quia unum circumstantia gignuntque echinum dodecaëdricum. In icosaëdro primo annuunt trina unum planum circumstantia: hinc ostrea icosaëdrica. Postea quina a quinis angulo coëuntibus dependentia: dicatur echinus major icosaëdricus, quinquelinearium angulorum; nondum tentavi constructionem. Anguli primum separantur, ubi ostreae anguli trilineares latent. Tertio, cum quina latera quina angulum claudentia connectant, non possunt esse omnia parallela, adnuunt igitur, et in eodem plano, quod est angulo substratum, continuata cum plano constituunt echinum minorem icosaëdricum, trilinearium angulorum.

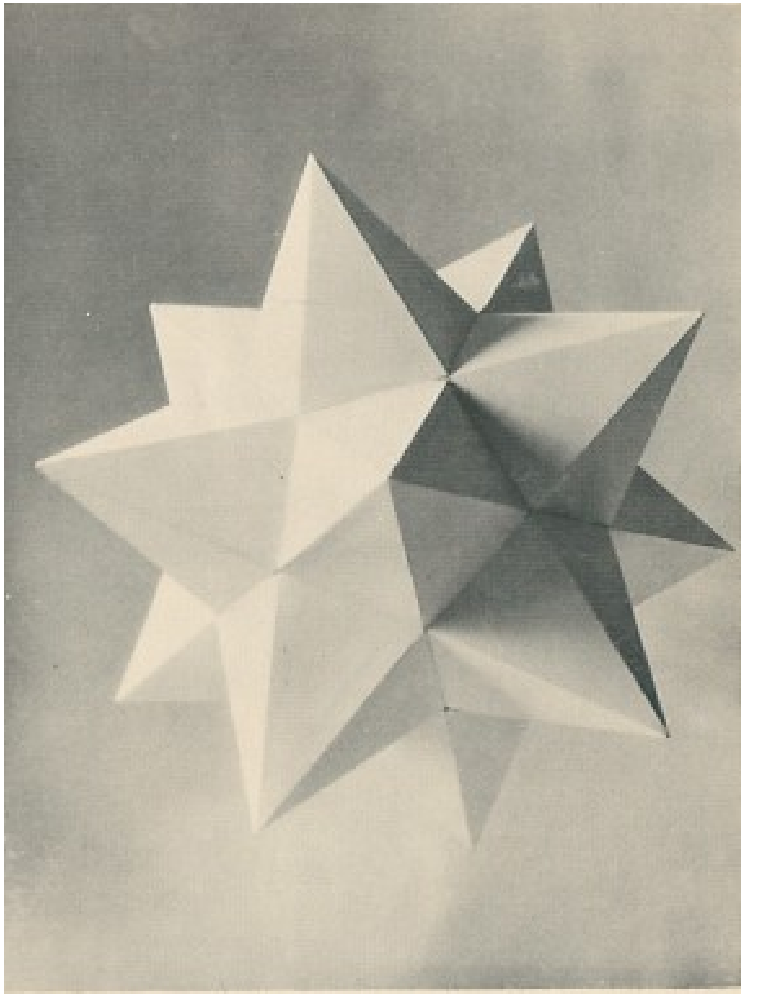
Search google (books) for Ostrea Kepler, find Joannis Kepleri astronomi opera omnia

Hugo Steinhaus: Mathematical Snapshots

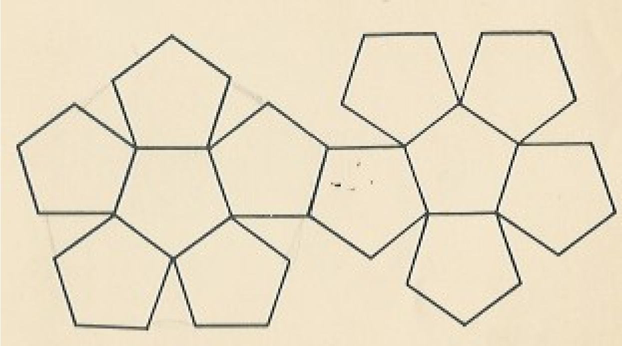
Stechert 1937

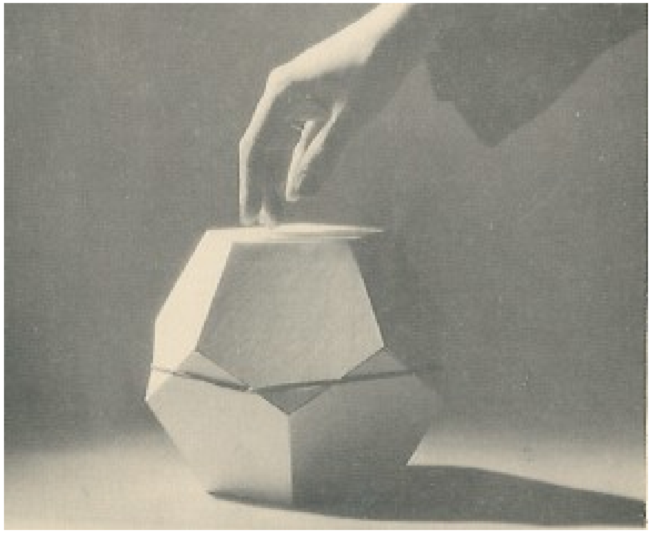
Oxford 1950

Dover 1999

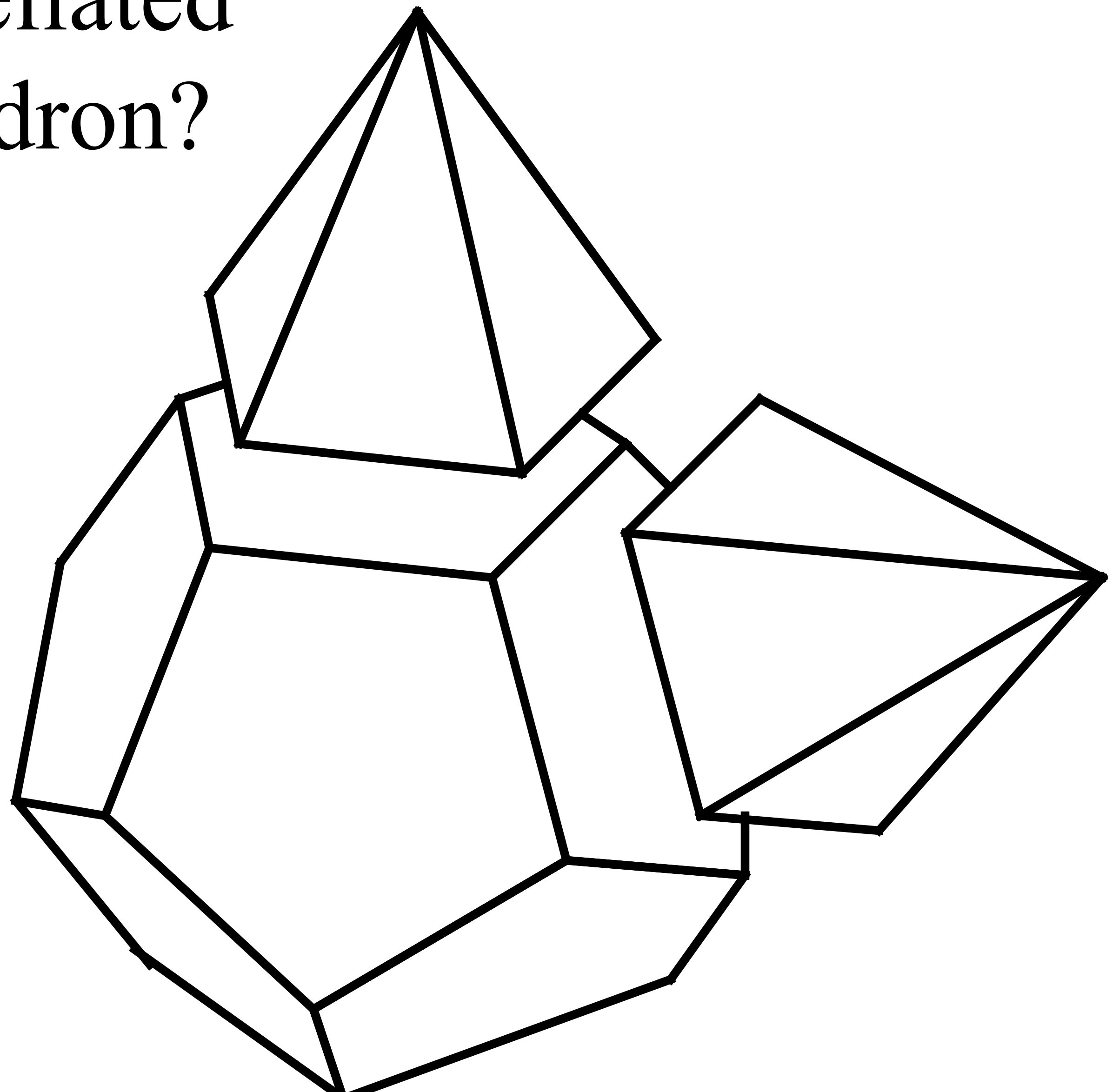


Build a dodecahedron from its net

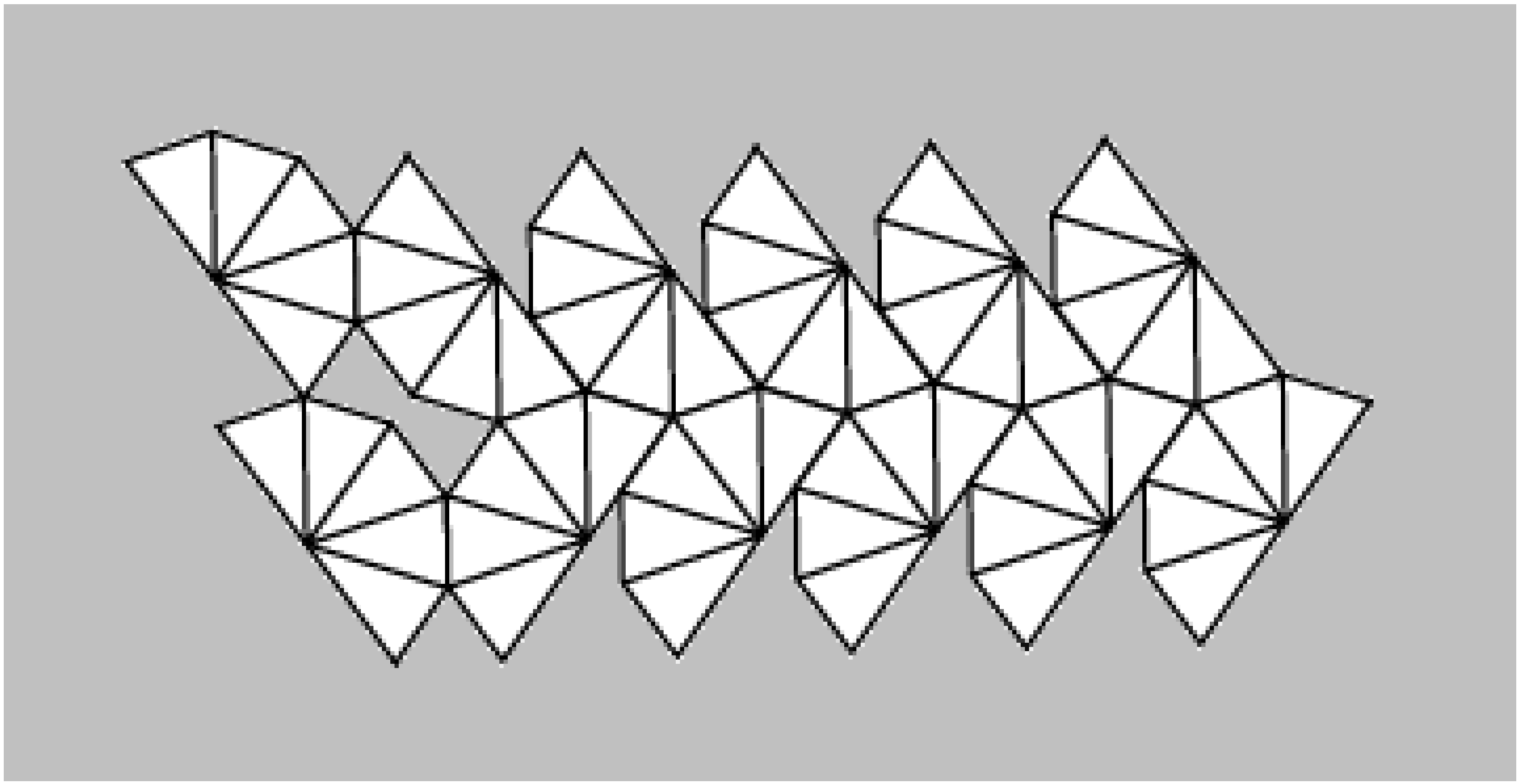




Build a stellated dodecahedron?

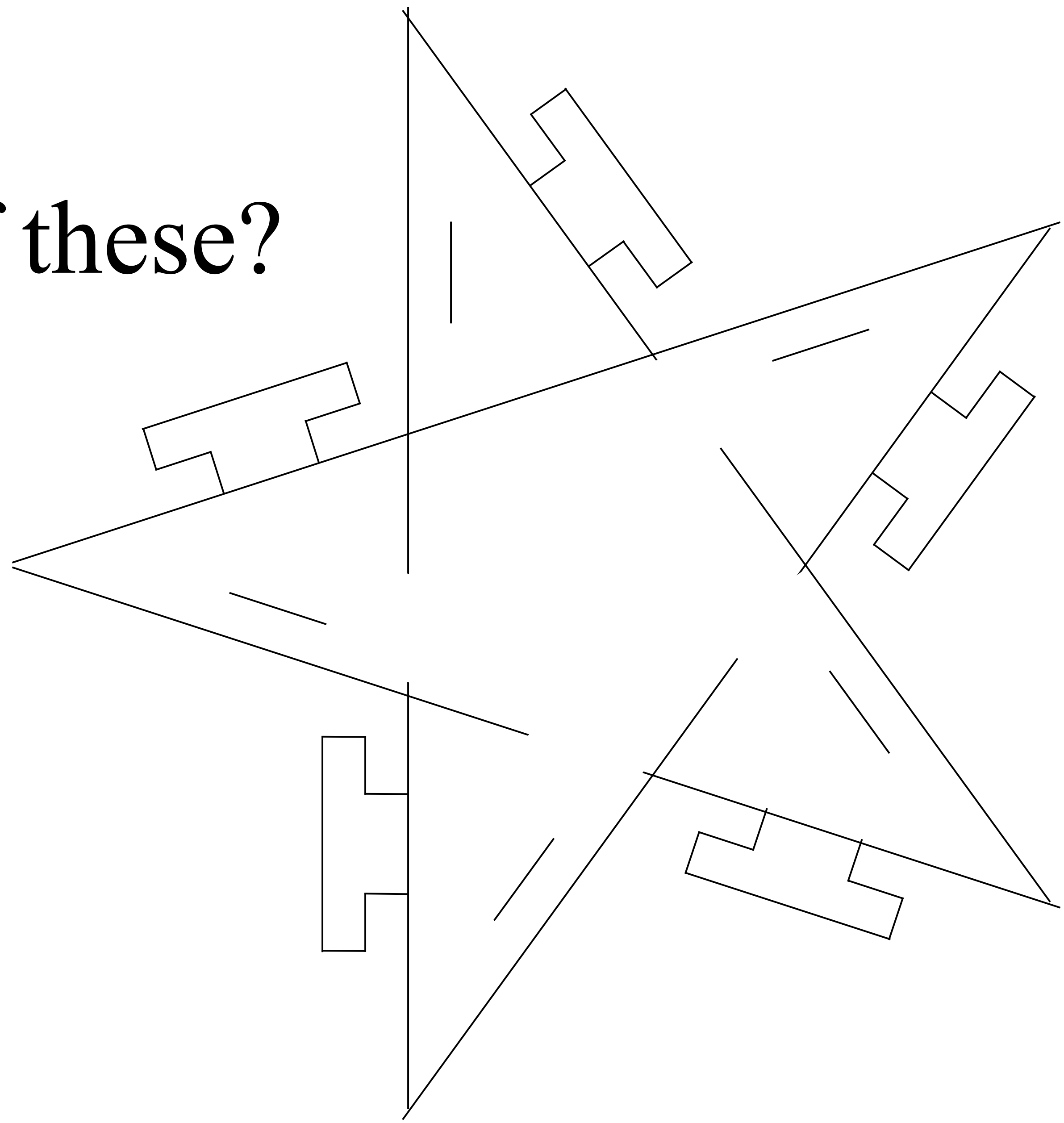


Net for the stellated dodecahedron

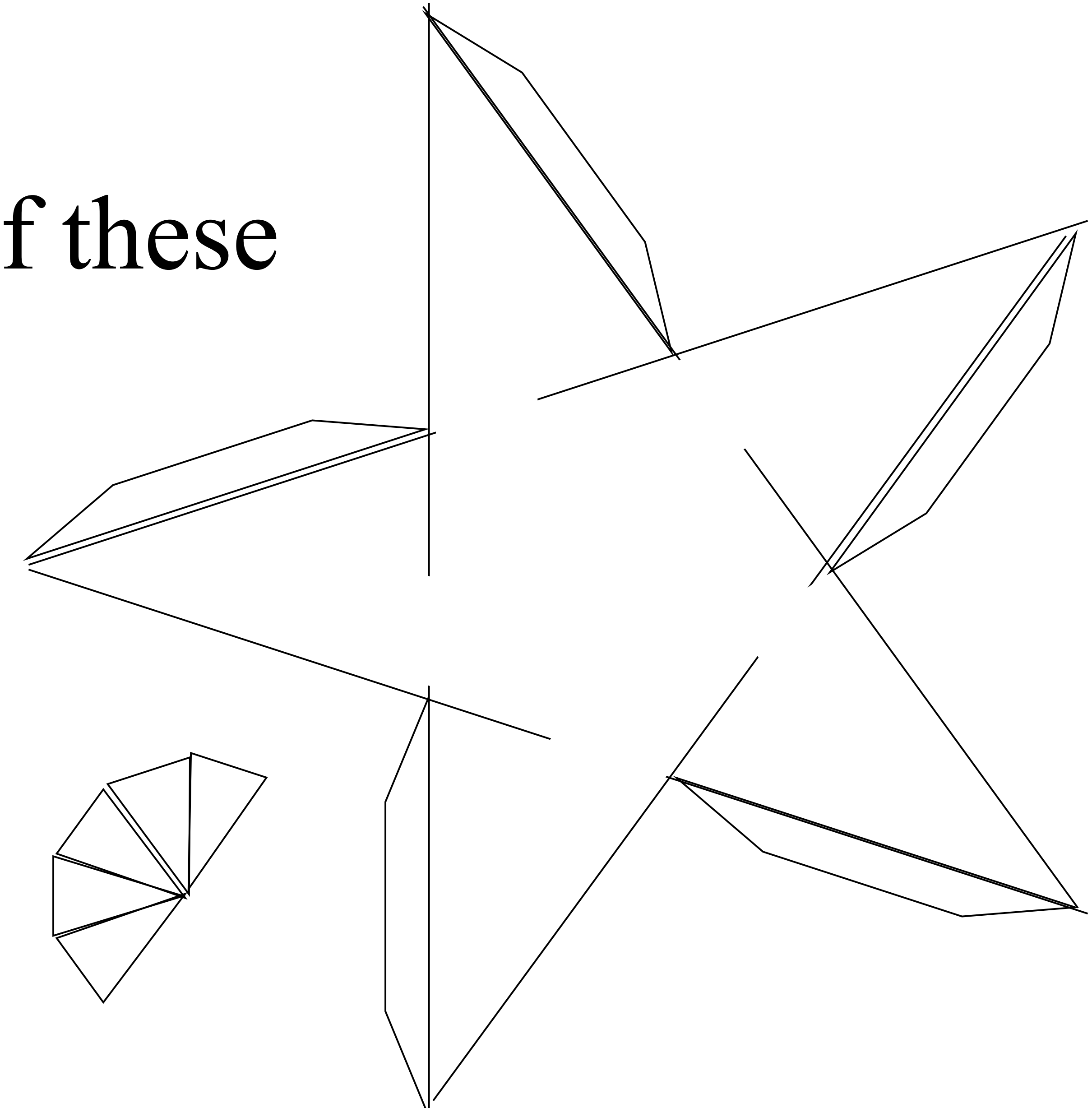


Cundy and Rollett, with construction tips

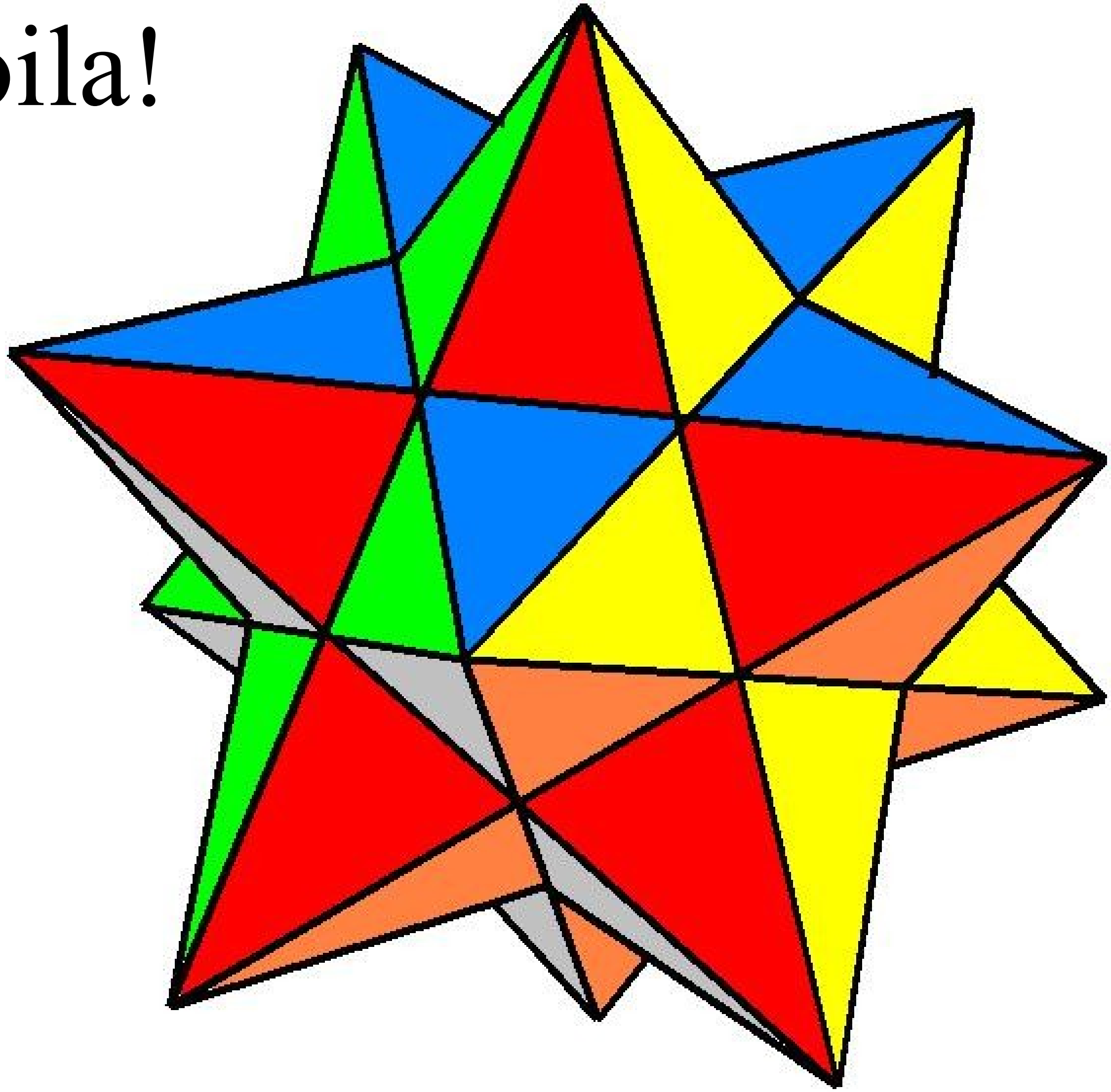
Use 12 of these?



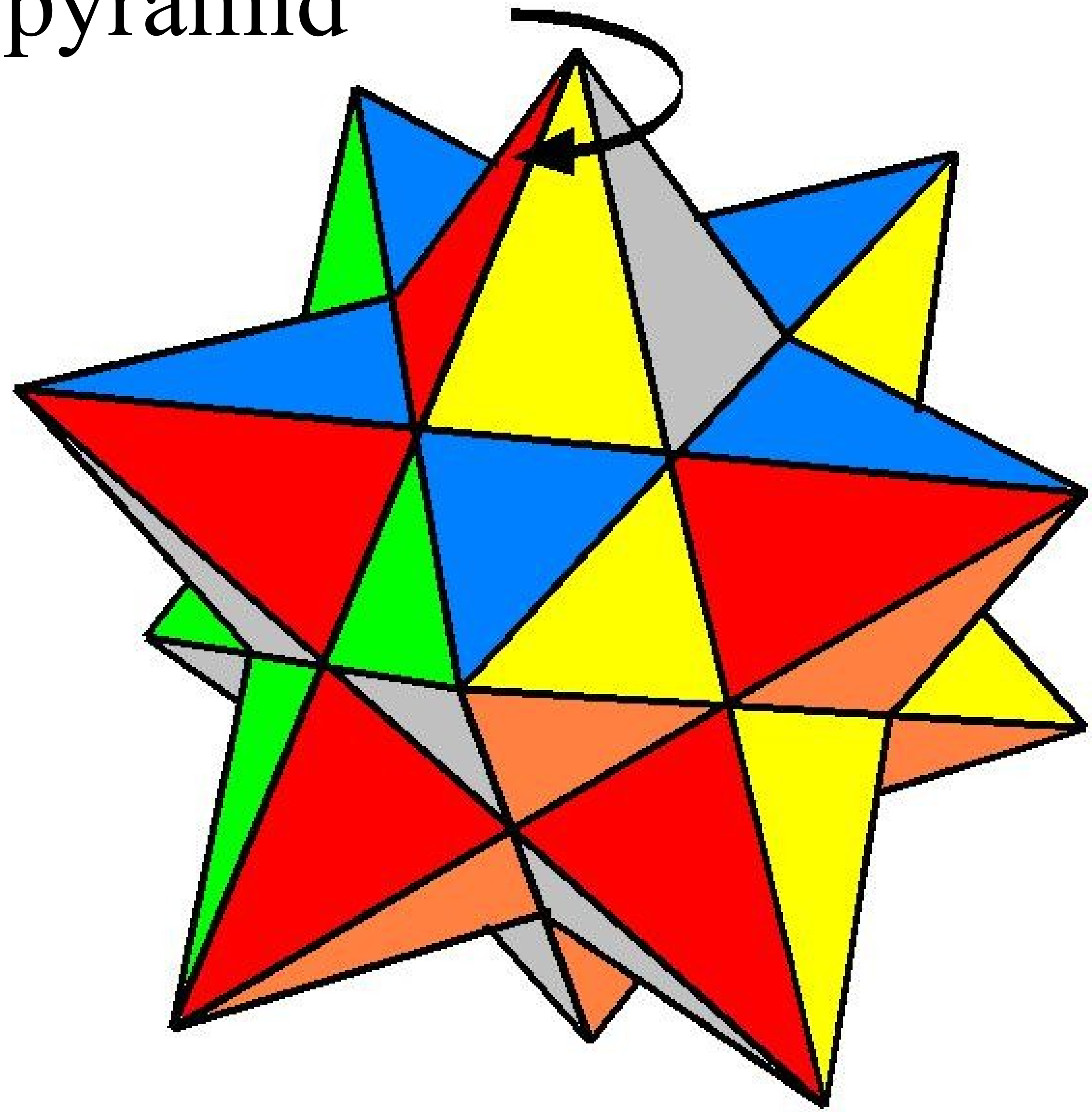
Use 12 of these



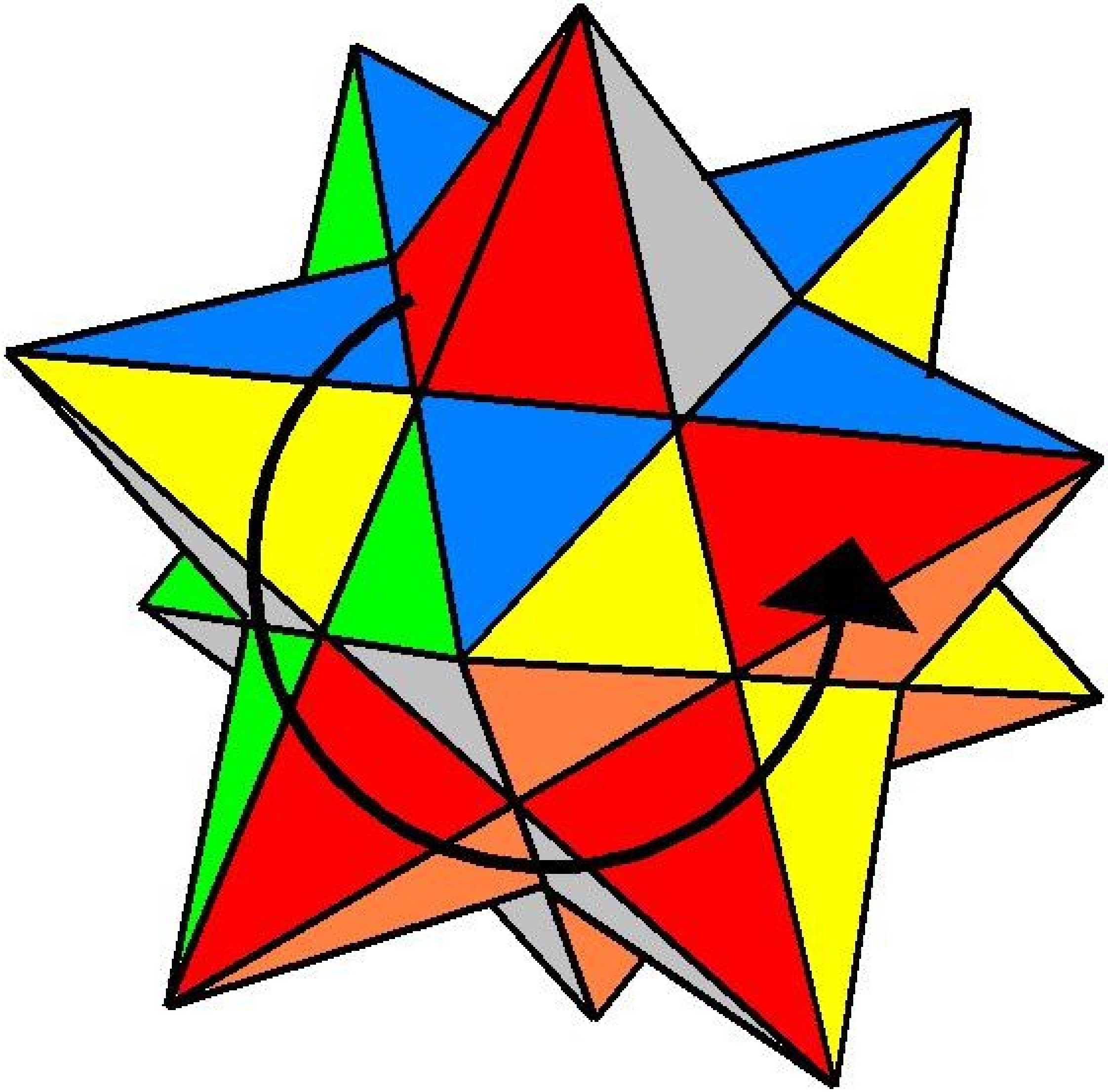
Voila!



Twist a pyramid

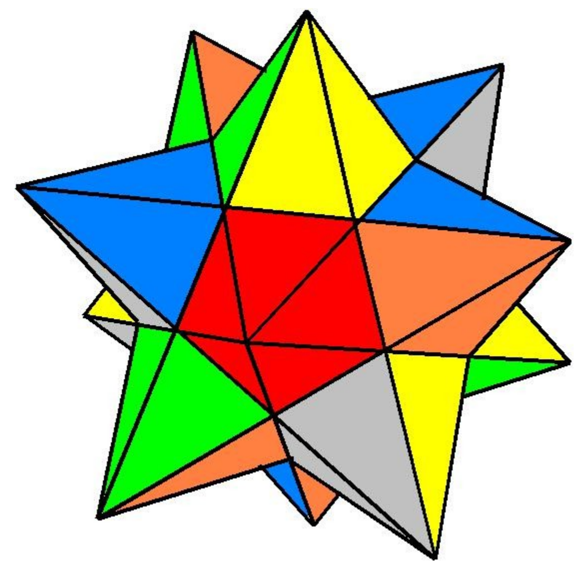


Twist a star



Rubik’s stellated dodecahedron

- Scramble, try to unscramble
- Harder than Rubik’s cube?
- What configurations
  are possible?
- Swap stars and pyramids?



Gadget vs. cube

- 6 colors
- 60 facets
- 6 axes
- 24 degree 5 moves
- Swap any pair of facets
- Pyramids (stars) commute, usually with each other too

- 6 colors
- 54 facets
- 3 axes
- 18 degree 4 moves
- Swap any pair of centers, edges, corners
- Only rotations about the same axis commute

The gadget group

- Acts as a group of permutations of the 60 facets
- Kernel has order $120 \times 10!^6 = 2.7 \times 10^{41}$
- Configuration space

  $$60!/|\text{kernel}| \sim 3 \times 10^{40}$$

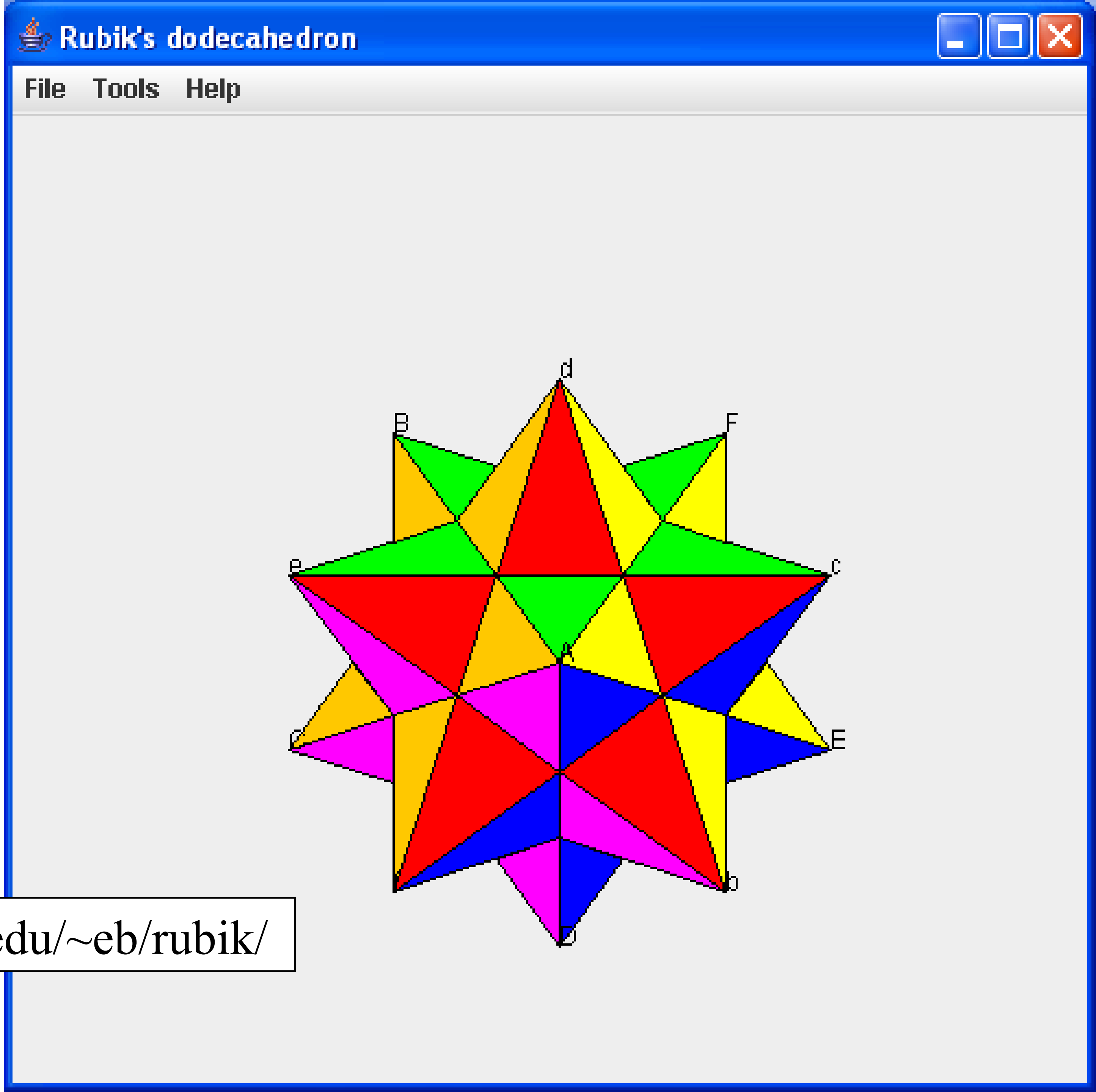
- Study action: orbits, macros

- David Joyner: Adventures in Group Theory: Rubik's Cube, Merlin's Machine, and Other Mathematical Toys

Can he build it?

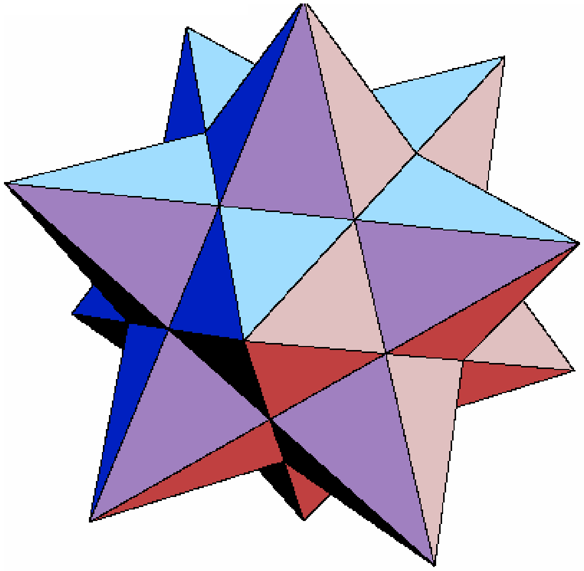
- Mechanical model is beyond my ability to imagine, let alone construct
- Try for virtual
- www.georgehart.com/virtual-polyhedra
- http://mathworld.wolfram.com/Kepler-PoinsotSo

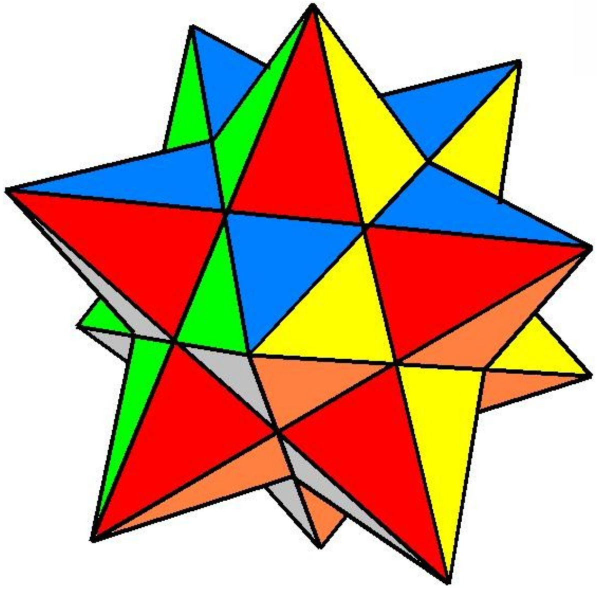
The virtual gadget

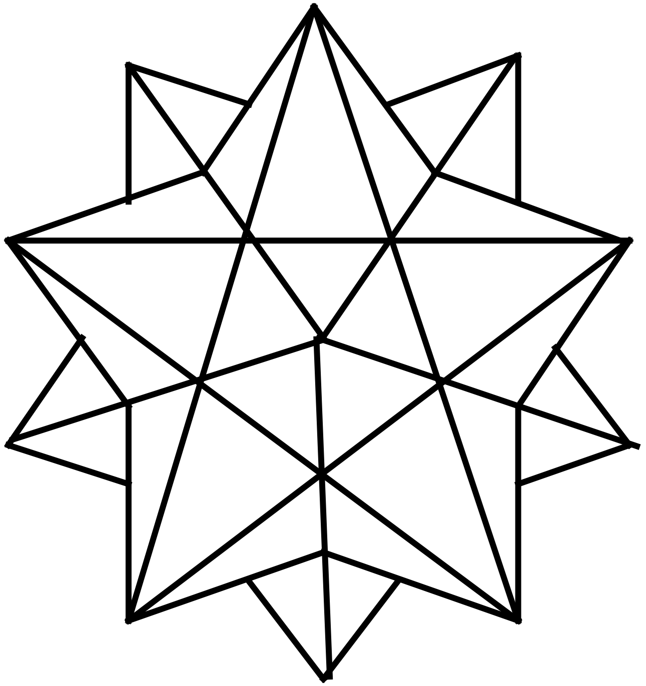


www.cs.umb.edu/~eb/rubik/

Projection





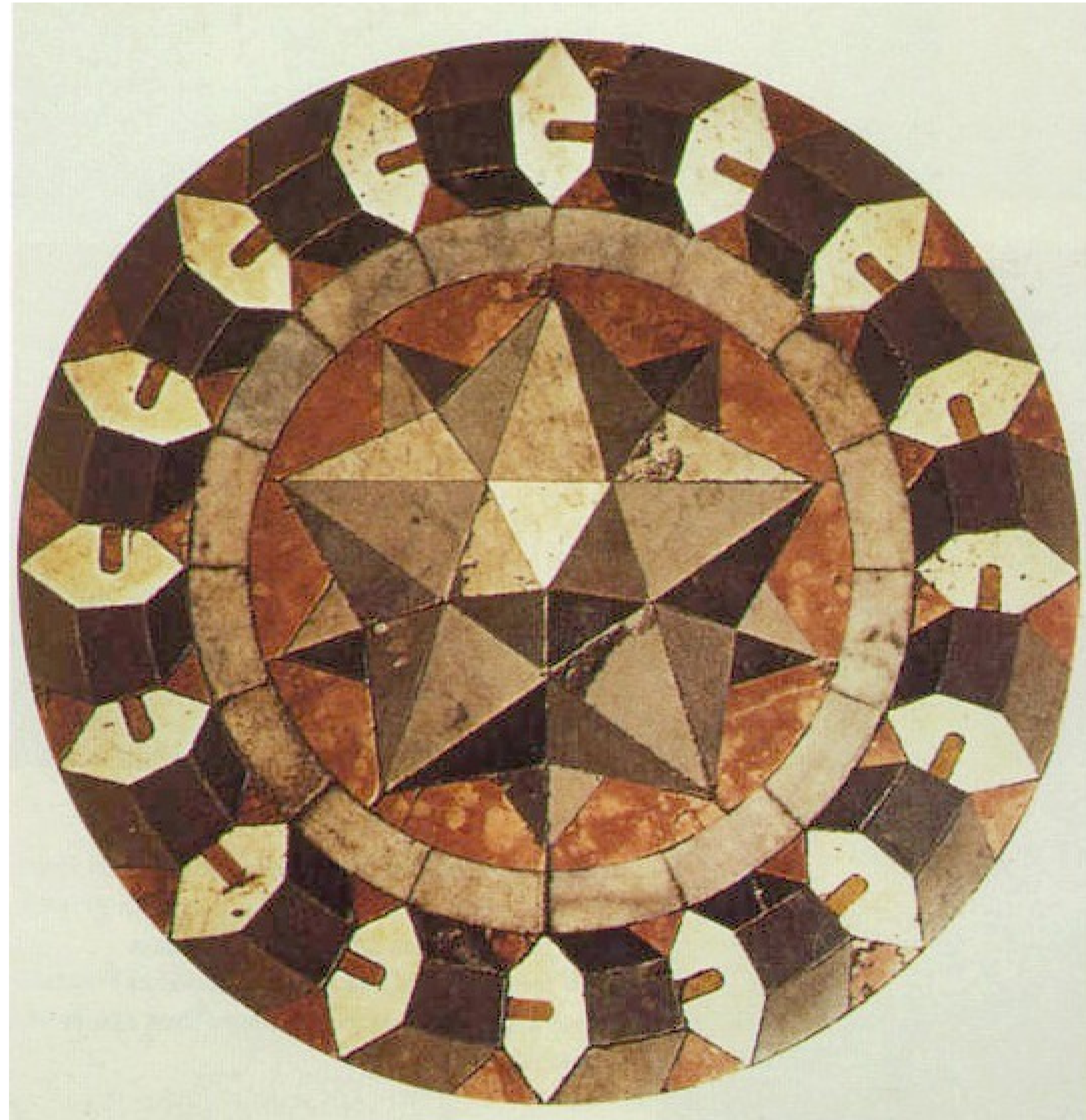


Paolo Ucello

Venice

Basilica of St. Mark

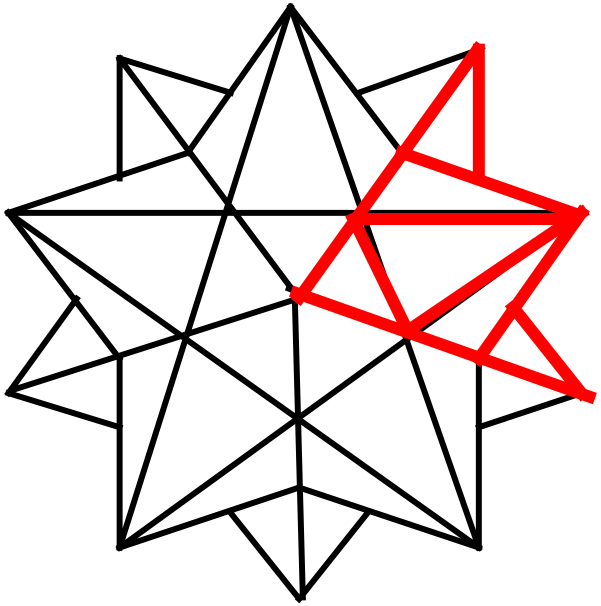
~1430



www.georgehart.com/virtual-polyhedra/uccello.html

Gadget.java

Geometry



Data structures …

Please help

- Work out the group theory
- Generalize
- Fix gadget mouse bug
- Build a real gadget